Auctions in the Post-Change-Order Period

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Abstract: A change order is frequently initiated by either the supplier or the buyer, especially when the contract is long-term or when the contractual design is complex. In response to a change order, the buyer can enter a bargaining process to negotiate a new price. If the bargaining fails, she pays a cancellation fee (or penalty) and opens an auction. We call this process the sequential bargaining-auction (BA). At the time of bargaining, the buyer is uncertain as to whether the bargained price is set to her advantage; indeed, she might, or might not, obtain a better price in the new auction. To overcome these difficulties, we propose a new change-order-handling mechanism by which the buyer has an option to change the contractual supplier after bargaining ends with a bargained price. We call this the *option mechanism*. By this mechanism, the privilege of selling products or services is transferred to a new supplier if the buyer exercises the option. To exercise the option, the buyer pays a prespecified cash payment, which we call the switch price, to the original supplier. If the option is not exercised, the bargained price remains in effect. When a switch price is proposed by the buyer, the supplier decides whether or not to accept it. If the supplier accepts it, the buyer opens an auction. The option is exercised when there is a winner in the auction. This article shows how, under the option mechanism, the optimal switch price and the optimal reserve price are determined. Compared to the sequential BA, both the buyer and the supplier benefit. Additionally, the option mechanism coordinates the supply chain consisting of the two parties. © 2015 Wiley Periodicals, Inc. Naval Research Logistics 62: 248–265, 2015

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1. INTRODUCTION

This article was motivated by our observation that after a buyer signs a contract with a supplier to procure products or services, frequently there is a *change order*, especially in cases where the contract is long-term or the contractual design is complex.

Long-term contracts in the coal, geothermal, uranium, and natural gas industries, for example, often include quantity and price provisions (Mulherin [16]). Change orders are initiated either at fixed dates (e.g., quarterly or annually) or at one party's request upon changes in labor, material, demand, or supply costs; the two parties then enter the bargaining process to negotiate new quantities and prices. In this way, long-term contracts are flexible in that prices and quantities can be adjusted to major market changes throughout the life of the contractual relationship. The literature on long-term-contractual quantity and price changes includes, among others, Mulherin [16] and Crocker and Masten [6] (natural gas contracts), Goldberg and Erickson [10] (petroleum coke contracts), and Joskow [14] (coal contracts between electric utilities and coal suppliers).

Change orders frequently occur also when the contractual design is complex. Unlike manufactured goods with standard characteristics (e.g., computers, washing machines, and DVD players), customized goods (e.g., new buildings, fighter jets, or consulting services) need to be tailored to meet the buyer's needs (Bajari et al. [3]). In such cases, it is highly likely that the buyer's desired specifications are incomplete,

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or otherwise subject to change, after contracts are signed (Bajari et al. [3] said "ex ante design is difficult to specify completely and ex post adaptations are expected"). Bajari and Tadelis [2] have found that both the buyer and the supplier share uncertainty with respect to many contingencies that can occur after a contract is signed, such as design failures, unanticipated site and environmental conditions, and changes in regulatory requirements. They offered the example of the Getty Center Art Museum in Los Angeles. The geological factors (canyons, slide planes, and earthquake fault lines) impacting on the project posed challenges for the team of architects and contractors. In fact, the project design had to be changed when contractors hit a slide and unexpectedly displaced 75,000 cubic yards of earth. And when an earthquake struck in 1994, cracks in the steel welds of the building's frame obliged the contractors to reassess the adequacy of the seismic design standards that had been followed. The project design also had to be altered due to the regulatory environment (Bajari and Tadelis [2]). Another relevant example is the recently opened Port Mann Bridge in British Columbia, Canada (Sinoski and Hoekstra [18]). A problem entailing ice accumulation was causing accidents, and a debate ensued as to whether the contractor should pay and/or undertake measures to mitigate this problem. The courts have recognized that when a change order to a fixed-price contract is initiated by a buyer in the construction industry, suppliers are entitled to compensation. In other words, without additional compensation, suppliers do not have an obligation to perform duties beyond those to which they are contractually bound (Bajari et al. [3]). Rather, once a change order occurs, the buyer bargains a new price with the supplier based on the changed terms. Any changes due to design failure, buyer priorities, goals, or other factors beyond the supplier's control will require a renegotiation of price.

Other examples of change orders can be found regarding Xerox Print Marketport, a print-job bidding system developed and sold as a service by Xerox. We have empirically observed that after auctions end, there are frequent change orders that result in a bargaining process. For example, suppose that a buyer wanted to outsource 10,000-copy print job for a specific future delivery date through an auction. However, after making the contract with the winner of the auction, the buyer realized that she needed fewer (say 9000) or more (say 12,000) copies. In this case, the buyer would initiate a change order and negotiates a new price with the supplier. Still other change orders could arise as well, since the time from the end of an auction to final delivery is often months long. Print shops could also receive, for example, new, rush orders during this period of time, in which case the print shop, as the supplier, might request a change order in order to change the delivery date, which would result in a bargaining process to negotiate a new price.

1.1. Possible Scenarios in the Post-Change-Order Period

Now assume that there is a contract between a buyer and a supplier, which in this case can be called the *primary supplier*, and that subsequently, there is a change order. The most frequent and typical process selected jointly by the buyer and the primary supplier, in Xerox Print Marketport (we have not observed any instances bypassing the bargaining process), is as follows. The parties bargain to negotiate a new price. If a price is agreed upon, a contract is signed accordingly. However, if there is no such mutually acceptable price, the buyer breaks the contract with the primary supplier (a penalty cost might be owed by the buyer to the primary supplier) and opens a new auction. As bargaining precedes the auction, we call this overall process the sequential bargaining-auction (BA).

Under the sequential BA, the buyer is uncertain if the bargained price is to her advantage, and wonders whether an actual price outcome (i.e., the *ex-post* price) of a new auction would be higher or lower than the bargained price. If she does not make a contract with the primary supplier through bargaining, she might be liable to pay not only the penalty cost for the contract cancellation but also the indirect cost. For example, a time delay could be a source of liability to the buyer. The buyer might also acquire a reputation for being difficult to work with, thus causing higher costs for future contracts. In the construction industry, the indirect cost will be significant if the buyer fails to find another supplier soon.

To overcome these uncertainties imposed on the buyer, we propose a systematic mechanism involving an option. The key idea is that an appropriately selected option enables the buyer to open a second auction without canceling the contract with the primary supplier, while the second auction enables her to check how the bargained price compares with the price she could get from other suppliers. To eliminate any confusion, we call an auction a *new auction* when the buyer opens an auction after canceling a contract with the primary supplier, and *a second auction* when she opens an auction while keeping the initial contract's bargained terms with the primary supplier. Under the option mechanism, the buyer and the supplier first bargain to negotiate a price. If the bargaining fails, the buyer pays a penalty cost to the primary supplier and opens a new auction. However, if the bargaining is successful and a price is agreed upon, the buyer is presented with an alternative: While keeping the bargained price with the primary supplier, offer a cash payment, which we call the *switch price*, to the primary supplier, which payment is given to the primary supplier when the option is exercised in exchange for the privilege of selling the products or services. The switch price can be considered as compensation for breaking the bargained contract.

If the primary supplier rejects the switch price, the buyer enters into a contract for the bargained price, and no further

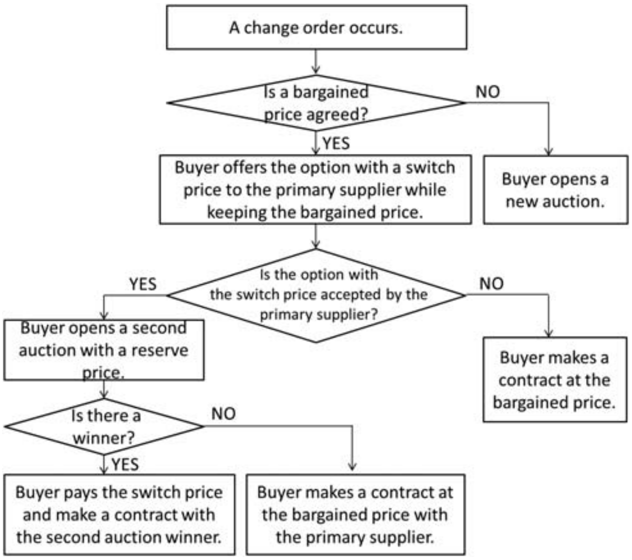

Figure 1. Option mechanism.

action is needed. If the primary supplier accepts the switch price, the buyer opens a second auction. The buyer proposes the switch price only once, and the primary supplier can either accept or reject that proposal (we will discuss this take-it-or-leave-it modeling choice later in this article). The switch price, as the compensation to the primary supplier for the buyer's breaking of the bargained contract, is larger than the penalty cost (this will be shown later in this article). The primary supplier receives a larger cancellation fee by allowing the buyer to have the bargained offer for an extended period of time. Note again that the option is a binding agreement that the two parties must honor. This means that neither party can back out of the option (e.g., there can be no renegotiation of the bargained price based on the result of the second auction). In the second auction with a reserve price, if there is a winner, the buyer exercises the option while paying the prespecified switch price to the primary supplier. By exercising the option, the buyer can purchase the products or services from the winner of the second auction. However, if there is no auction winner (i.e., if there are no bids below the reserve price), the buyer makes a contract with the primary supplier at the bargained price. The option mechanism is illustrated in Fig. 1. We will show how the optimal switch price and the optimal reserve price (for the second auction) are determined under the option mechanism.

We will then compare the option mechanism with the sequential BA. We show that the option mechanism is beneficial for both the buyer and the primary supplier. The specific benefit comes from the other bidders (in the second auction), and is split between the buyer and the primary supplier in the bargaining phase. Therefore, the benefit to the buyer (or the primary supplier) is proportional to her (or his) bargaining power. Once the option is accepted by the primary supplier, it enables the buyer, as noted earlier, to screen other suppliers to find a better price, without canceling the contract with the primary supplier. The benefit of the option mechanism over the sequential BA is especially significant when the cost of the primary supplier is neither too high nor too low in the cost range determined by the auction market. At this point, the buyer has the greatest uncertainty about whether the primary supplier has a higher or lower cost than the other suppliers in the auction. Additionally, we show that the option mechanism coordinates the supply chain consisting of the buyer and the primary supplier. In other words, by coordinating the two parties, it generates the maximum surplus.

Note that the buyer needs to perform an economic analysis before issuing a change order, especially when the new requirement can be subdivided as in the Xerox example. Assume for example that the buyer needs 12,000 copies instead of 10,000, and that the penalty cost is enormous. In this case, the buyer could keep the original contract with the primary supplier for 10,000 copies while outsourcing 2000 copies from another vendor. If the expected profits under the sequential BA and the option mechanism after the issuing of the change order are less than those under the alternative of keeping the current contract and finding another supplier without issuing any change order, the buyer certainly should not issue the change order. The economic analysis is beyond the scope of this article. The model presented in the article is relevant under the following business-environment conditions: (1) the penalty cost is not prohibitively expensive; (2) the task cannot be easily subdivided; or (3) the subdivision has a nonlinear cost impact (e.g., a quantity discount, according to which it is cheaper to procure 12,000 copies than 10,000 and 2000 separately) or large fixed costs (e.g., the total delivery cost from two different suppliers is significant).

We briefly note here an alternative means of handling a change order. The buyer could open an auction first in order to find a new supplier, before attempting to negotiate with the primary supplier; this we call sequential auction-bargaining (AB). This practice might result in the buyer receiving a bad reputation. And consequentially, the primary supplier might choose not to return to the bargaining table after the auction. Moreover, it is not clear what the buyer is committing to in the auction; in other words, the winner of the auction might or might not end up supplying the buyer. If the buyer reneges on her commitment to the winner of the auction, her reputation could be negatively affected. Perhaps not surprisingly then, we have not come across any actual implementation of the sequential AB in practice. Nonetheless, in Appendix A, we carry out a detailed analysis of the sequential AB for comparison with the option mechanism. We show that the option mechanism, relative to the sequential AB, is beneficial for the buyer. Thus, significantly, there is no incentive for the buyer to open an auction in advance of the negotiation if the option mechanism is available. Another consideration in this regard is the fact that the sequential AB does not coordinate the supply chain consisting of the buyer and the primary supplier, whereas the option mechanism does.

The scope of our research encompasses the handling of a change order when the buyer and the primary supplier have sufficient information on each other (i.e., perfect information) as the result of an established long-term relationship. The unique contribution of our work is our derivation and proposal herein of what we call the option mechanism: in effect, a new change-order-handling mechanism. To our best knowledge, nothing like the option mechanism has been applied in practice to handle a change order. By demonstrating the advantages of the option mechanism relative to the alternatives, we build an argument for this mechanism as a practical solution to the difficulties incurred in the issuance of change orders.

1.2. Literature Review

We propose the option mechanism that combines auctions and bargaining. In this regard, there are several papers analyzing combined auction and bargaining models. Some papers (e.g., Bulow and Klemperer [4], Elyakime et al. [7]) combine them such that an auction precedes bargaining while Engelbrecht-Wiggans and Katok [8] combine them such that bargaining precedes an auction. However, those papers do not consider change orders in their models. There are a few papers (e.g., Bajari and Tadelis [2], Bajari et al. [3]) dealing with change orders in the postauction period, but they do not consider the possibility of changing suppliers. In a different domain of the airline revenue management, the paper by Gallego et al. [9] studies the option to change buyers. We study the option model to change suppliers in the procurement environment using auctions and bargaining.

Bulow and Klemperer [4] consider the second-price auction to choose a winning bidder in the first phase and use ultimatum take-it-or-leave-it offers in the second phase. This article shows that the auctioneer has greater expected profit in this combined model than in the auction-only model. Bulow and Klemperer [4] use the ultimatum bargaining model with incomplete information, but we use the bargaining model incorporating the relative bargaining power of the buyer and the supplier with complete information. In addition, they consider the bargaining phase in the second phase to acquire a lower price while ignoring the possible change orders. We explicitly consider the change order and propose the option mechanism that is beneficial for both the buyer and the supplier compared to the standard bargaining mechanism.

Elyakime et al. [7] study the first-price auction followed by the sequential alternating bargaining with complete information. In the auction phase, the auctioneer submits a secret reserve price and if there is no bid meeting the reserve price, a second phase of bargaining takes place. They provide an equilibrium bidding strategy that is a solution to a first-order differential equation and show numerically that both the auctioneer and the bidders prefer the model to the auction-only model. This article does not consider the change order either.

The first-price auction model followed by the sequential alternating bargaining with one-side uncertainty is proposed by Wang [20]. The bargaining is one-side incomplete because the winning seller's cost becomes known but the buyer's valuation is private. He provides an equilibrium bidding strategy that is a solution to a first-order differential equation.

The sequential auction-bargaining model is also studied by Huh and Park [13]. While the above papers (Elyakime et al. [7] and Wang [20]) provide an equilibrium bidding strategy that is a solution to a first-order differential equation, Huh and Park [13] provide an equilibrium bidding strategy in a closed form while assuming the bargaining is in the sequential alternating model with complete information. Their assumption of complete information is applicable if the value and cost information can be accurately estimated or if the information is equally uncertain to both the buyer and the supplier. They find that the buyer sets a less aggressive reserve price in the auction-bargaining model than in the standard auction-only model and show that a rational buyer prefers the second-price auction and bargaining model to the first-price auction and bargaining model.

While an auction precedes bargaining in all the above models, Engelbrecht-Wiggans and Katok [8] consider a model where bargaining precedes an auction. The buyer procures multiple products from suppliers who each can produce one unit. In the first phase, some suppliers are chosen to provide one unit. The price is determined in the second phase based on the rule of the general second-price auction where all the suppliers in the first phase are excluded. In our model, the second auction follows the bargaining process as their model. However, the main difference is that they consider the second-phase auction to determine the price, but we consider the second auction to switch suppliers.

All of the above papers mentioned so far demonstrate the benefits of combining auctions and bargaining to have a lower price. However, there are few studies that deal with change orders that entail the price renegotiation. Bajari and Tadelis [2] develop a model explaining the tradeoff between the cost of providing a more comprehensive design and the cost of negotiation. More complete design covers more future events that may happen. The probability of the occurrence of change orders is proportional to the number of nonconsidered future events. Once a change order is initiated, the costly negotiation process follows. Therefore, the costly negotiation can be avoided by providing more comprehensive design. However, it is also costly for the buyer to provide a more comprehensive design. They show how the buyer can optimally set the design completeness parameter endogenously. Bajari et al. [3] show that auctions may perform poorly when projects are complex and contractual design is incomplete. Therefore, there are frequent change orders after an auction ends. Even though the two papers consider the frequent change order cases, they did not consider the possibility of switching suppliers as we do in this article.

Gallego et al. [9] apply the option to the airline revenue management problem where low-fare customers come before high-fare customers. Each low-fare customer can choose to purchase either a standard low-fare product or a callable product with a prespecified recall price. The prespecified recall price is paid to the low-fare customer whose option is called later, and a low-fare customer is switched to a high-fare customer. The option is exercised when the number of high-fare booking exceeds the remaining capacity after the low-fare sales. It is possible to consider that the prespecified recall price is determined in the ultimatum game by the following reasons. First, the price is published by the airline. Second, if the price is acceptable to customers, they buy the callable product. Otherwise, they buy the regular one. Similarly, the switch price in our model is determined in the ultimatum game. When the switch price is proposed by the buyer, the primary supplier accepts it if the price is attractive to him. Otherwise, the contract is made at the bargained price. We study how the switch price, the bargained price, and the reserve price are determined in the procurement environment using bargaining and auctions.

1.3. Overview

First, we analyze the sequential BA in Section 2. We then describe our model of the option mechanism in more detail and show how the buyer can set the optimal switch price and the optimal reserve price for the second auction in Section 3. In Section 4, we compare the option mechanism with the sequential BA. In Section 5, we model the general bargaining game for the switch price (a variant of the option mechanism) that includes the ultimatum game as a special case. We then show that the switch price is optimally determined for the two parties when the buyer makes a take-it-or-leave-it offer as studied in Section 3. We present the conclusion in Section 6. We also analyze the sequential AB and compare it with the option mechanism in Appendix A.

2. SEQUENTIAL BA

In this section, we analyze the sequential BA to deal with change orders. Under the sequential BA, the buyer and the primary supplier negotiate a new price based on the changed terms in Phase 1. If there is an agreed price, then the buyer and the primary supplier make a contract at the agreed price (i.e., the bargained price). However, if there is no such a price, the buyer breaks the contract with the primary supplier while paying the penalty cost and opens a new auction in Phase 2. We first consider the new auction in the second phase assuming bargaining has failed in the first phase. We then go back to the analysis of bargaining.

2.1. Phase 2: New Auction

Assume that bargaining has failed in the first stage and the buyer opens a new auction. The auction mechanism is either the first-price or the second-price procurement auction. In a first-price auction the lowest bidder is the winner and the price is his bid, whereas in a second-price auction the lowest bidder is the winner and the price is the second-lowest bid. Because the changed terms could not be negotiated with the primary supplier through the bargaining process (possibly due to the unusual high cost of the primary supplier or the primary supplier's inability to deliver the changed terms), we assume that the buyer (or the procurement system) prevents the primary supplier from bidding in the new auction.

Assume that there are n bidders (not including the primary supplier, who is excluded in the auction) if the buyer opens a new auction to purchase the changed products or services. The buyer has valuation v when the changed terms are delivered. We consider the private value model where bidders' production costs are independent and identically distributed. Bidders are indexed by $i = 1, 2, \ldots, n$. Under the private value model, the auction has been well analyzed in literature (for example, Krishna [15]). Let C_i represent a random variable of the production cost of bidder i with the support of $[\underline{c}, \bar{c}]$, where $\underline{c} \geq 0$ and $\bar{c} < \infty$. Let $F(\cdot)$ and $f(\cdot)$ represent the cumulative distribution function and the probability density function of each C_i. We assume that $f(c) > 0$ for all $c \in [\underline{c}, \bar{c}]$. We define a random variable Y_i to denote the minimum cost among i bidders, that is, $Y_i \sim \min\{C_1, \ldots, C_i\}$. Let $G_i(\cdot)$ and $g_i(\cdot)$ denote the cumulative distribution function and the probability density function of Y_i. Also let $\bar{F}(\cdot) = 1 - F(\cdot)$ and $\bar{G}_i(\cdot) = 1 - G_i(\cdot)$. We have $\bar{G}_i(c) = \bar{F}(c)^i$ and $g_i(c) = i \cdot \bar{F}(c)^{i-1} \cdot f(c)$. An indicator function is denoted by $I\{\cdot\}$.

We review the results from the classical auction literature about the symmetric bidding strategy (i.e., all bidders follow the same strategy) (Krishna [15]). In the first-price procurement auction with reserve price r, the symmetric bidding strategy of a bidder with opportunity cost c is $E[\min\{Y_{n-1}, r\} | Y_{n-1} > c]$ if $\underline{c} \leq c \leq r$; and if $c > r$, then it is ∞, that is, not to submit any bid. In the second-price procurement auction with reserve price r, the symmetric bidding strategy of a bidder with opportunity cost c is to bid c. The probability of having a winning bid is $Pr(Y_n \leq r) = G_n(r)$.

Even though the first-price auction has a different bidding strategy from the second-price auction, the buyer's expected profits are the same, which is called the Revenue-Equivalence

Theorem (Krishna [15], Section 6.2.4 of Talluri and van Ryzin [19]). We state the results in the following Lemma 2.1. We define $P_A(r)$ as the random amount of payment of the buyer and $\tilde{P}_A(r)$ as the random conditional amount of payment given that there is a winner. Under these definitions, $P_A(r) = \tilde{P}_A(r) \cdot I\{Y_n \le r\}$. In the first-price or the second-price procurement auction with reserve price r, the expected payment of the buyer is

$$E[P_A(r)] = E[\min\{Y_{n-1}, r\} \cdot I\{Y_n \le r\}]$$
$$= n \cdot E[E[\min\{Y_{n-1}, r\} \cdot I\{Y_{n-1} > C_n\} | C_n] \cdot I\{C_n \le r\}].$$

Note that

$$E[P_A(r)] = E[\tilde{P}_A(r)] \cdot G_n(r).$$

Throughout the article, we assume that $F(\cdot)/f(\cdot)$ is increasing to assure the optimal reserve price, which is a standard assumption in the auction optimal mechanism design literature.

LEMMA 2.1 (Krishna [15]). In the first-price or the second-price procurement auction with reserve price r, the expected profit of the buyer is

$$\Pi_A(r) = G_n(r) \cdot v - E[P_A(r)]$$
$$= G_n(r) \cdot (v - E[\tilde{P}_A(r)]). \quad (1)$$

Furthermore, the optimal reserve price of r^* maximizing $\Pi_A(r)$ is the solution r of the following equation

$$v - r = \frac{F(r)}{f(r)}. \quad (2)$$

The above lemma and its proof are given in Section 2.5 of Krishna [15]. We denote the optimal expected profit of the buyer from opening the new auction as

$$\Pi_A^* = \Pi_A(r^*). \quad (3)$$

2.2. Phase 1: Bargaining

When a change order is initiated, the buyer and the primary supplier enter the bargaining process to negotiate a new price for the changed terms. We use the general bargaining model (Elyakime et al. [7] and Huh and Park [13]) for this phase. The buyer has valuation v, and the primary supplier has cost c_p to deliver the changed terms, where $v \ge c_p$. The bargaining game is between the buyer with valuation v and the primary supplier with cost c_p. Let $\lambda \in [0, 1]$ be the bargaining power of the buyer. We assume that v, c_p, and λ are all public information. The penalty cost that the buyer needs to pay to the primary supplier when the contract breaks (i.e., the negotiation fails) is denoted by K. The penalty cost is predetermined when the original contract is signed.

Table 1. Payoffs depending on bargaining outcome under sequential BA

Bargaining outcome	Buyer's payoff	Primary supplier's payoff	Sum
Failure	$\Pi_A^* - K$	K	Π_A^*
Success	$v - p_Q$	$p_Q - c_p$	$v - c_p$
Contract value (Diff.)	$v - p_Q$ $+K - \Pi_A^*$	$p_Q - c_p - K$	$v - c_p - \Pi_A^*$

We have used the private information model for the costs of all the new bidders in the auction phase but the perfect information model for the cost of the primary supplier in the bargaining process. The perfect information assumption is justifiable when the two parties are in a long-term relationship. In this case, the buyer can estimate the cost of the supplier because the changed order is typically not too different from the original order. Meanwhile, the cost estimation is difficult to do for all the new bidders in the auction stage (there will be many bidders and very little information on them), and thus we use the private information model for the auction. Also if the cost is proportional to the quantity of the order and there is a change in quantity only, then the buyer can easily know the cost of the primary supplier for the change.

If bargaining ends with failure, the buyer pays penalty cost K to the primary supplier and opens a new auction in the second phase. If a price is agreed, they make a contract at the bargained price that is denoted by p_Q. Table 1 summarizes the payoffs of the buyer and the primary supplier depending on the result of bargaining. If the sum of failure payoffs is greater than the sum of success payoffs (i.e., $\Pi_A^* > v - c_p$), then no bargained price exists and the buyer searches for a new supplier by opening a new auction. However, if $\Pi_A^* \le v - c_p$, the bargained price is determined using the bargaining powers of the two parties. As the bargaining power of the buyer is λ and that of the primary supplier is $1 - \lambda$, the outcome of the contract value for the buyer is $\lambda \cdot (v - c_p - \Pi_A^*)$ and that for the primary supplier is $(1 - \lambda) \cdot (v - c_p - \Pi_A^*)$. In the generalized Nash bargaining framework with the relative bargaining powers, the bargained price is determined such that the ratio between $v - p_Q + K - \Pi_A^*$ and $p_Q - c_p - K$ is $\frac{\lambda}{1-\lambda}$ (Nagarajan [17]). The bargained price is stated in the following theorem.

THEOREM 2.1: Under the sequential BA, if $\Pi_A^* \le v - c_p$, the bargained price is determined as

$$p_Q = \lambda \cdot c_p + (1 - \lambda) \cdot (\bar{G}_n(r^*) \cdot v + E[P_A(r^*)]) + K. \quad (4)$$

The bargained price is greater than penalty cost K if it exists. The buyer has to pay the contract price that is higher than the penalty cost to the primary supplier. This makes sense since the buyer shares the gains of bargaining with the primary supplier. Furthermore, the bargained price is linearly increasing in penalty cost K, which means that the buyer needs to pay more contract price when the penalty cost is higher. The bargained price can also be considered to include the penalty cost. In summary, under the sequential BA, the expected profit of the buyer is

$$\Pi_Q = \begin{cases} \lambda \cdot (v - c_p) + (1-\lambda) \cdot \Pi_A^* - K & \text{if } v - c_p \geq \Pi_A^* \\ \Pi_A^* - K & \text{if } v - c_p < \Pi_A^*, \end{cases} \tag{5}$$

and the expected profit of the primary supplier is

$$U_Q = \begin{cases} (1-\lambda) \cdot (v - c_p - \Pi_A^*) + K & \text{if } v - c_p \geq \Pi_A^* \\ K & \text{if } v - c_p < \Pi_A^*. \end{cases} \tag{6}$$

3. OPTION MECHANISM

Assume that there is a change order after the original contract is signed and the option mechanism is used to make a contract. The option mechanism has the following three phases: (1) The buyer and the primary supplier enter a bargaining process to have a bargained price. If bargaining fails, the buyer opens a new auction after paying the penalty cost to the primary supplier. If there is an agreed bargained price, then it goes to Phase 2. (2) The buyer offers the option with switch price p_{sw}. The ultimatum take-it-or-leave-it game is used. If the primary supplier rejects the switch price, they make a contract at the bargained price and no further action is needed. However, if the primary supplier accepts the switch price, then it goes to Phase 3. (3) The buyer opens a second auction with a reserve price. If there is an auction winner, the buyer exercises the option and pays the switch price to the primary supplier. The buyer purchases the products or services from the winner of the second auction. However, if there is no auction winner (i.e., no bids below the reserve price), the buyer makes a contract with the primary supplier at the bargained price.

For the switch price, we consider the ultimatum game because it is easier to understand the mechanism than the bargaining game with relative bargaining powers. In Section 5, we study the general bargaining game for the switch price that includes the ultimatum game as a special case. We show that it is beneficial for the two parties to use the ultimatum game where the buyer makes a take-it-or-leave-it offer compared to the bargaining game with relative bargaining powers. Furthermore, we believe it is reasonable to model it as the ultimatum game since the buyer and the primary supplier have already agreed on the bargained price at the end of Phase 1, and this is an additional offer that the buyer can propose.

In the bargaining phase (Phase 1), the two parties compare their expected profits at the bargained price and those from the failure of bargaining. Under the option mechanism, the profits of the contract at the bargained price depend on (1) the switch price that determines the supplier decision to accept the option terms and (2) the future uncertain event of exercising the option. We compute the bargained price under the option mechanism, denoted by p_O. If they do not have an agreed price through bargaining, the contract breaks and the buyer pays the penalty cost to the primary supplier. However, if the two parties reach bargained price p_O, then they make a contract at p_O and go to Phase 2.

In the option phase (Phase 2), the buyer offers a switch price, denoted by p_{sw}, to the primary supplier. If the switch price is rejected, the bargained price is in effect and the expected profit of the primary supplier is $p_O - c_p$. The rational primary supplier accepts the switch price when his expected profit is greater than or equal to $p_O - c_p$ (the option participation constraint). We assume that he accepts the option terms at the tie (i.e., his expected profit is $p_O - c_p$). Under this assumption, the optimal switch price p_{sw}^* is determined (see Theorem 3.1). If the primary supplier accepts the switch price, they go to Phase 3.

In the second auction phase (Phase 3), the buyer opens a second auction with a reserve price. The mechanism can be either the first-price or the second-price procurement auction. We denote the expected profit of the buyer in Phase 3 as $\pi_O(r; p_O)$ where r is the reserve price and show how to calculate the optimal reserve price of r_O^* maximizing $\pi_O(r; p_O)$.

As the bargaining process precedes the option and the second auction, we consider the option and the second auction first. The bargained price depends on the future switch price and second auction price. Therefore, we first determine optimal switch price p_{sw}^* and optimal reserve price r_O^* assuming that the bargained price is given by p_O. We then go back to the bargaining process to determine *po*.

3.1. Phase 2 and Phase 3: Option, Switch Price, and Second Auction

We show how to set the optimal switch price, denoted by p_{sw}^*, and the optimal reserve price for the second auction, denoted by r_O^*. We assume that the primary supplier is rational and accepts the option terms with a switch price at which his expected profit is greater than or equal to $p_O - c_p$ when the bargained price in Phase 1 is p_O (the option participation constraint). When the primary supplier accepts the option, the profit of the primary supplier is $p_O - c_p$ if the option is not exercised and p_{sw} if the option is exercised. If $p_{sw} < p_O - c_p$, his expected profit is less than $p_O - c_p$ and he does not accept the switch price. Therefore, the lowest accepted switch price is $p_O - c_p$. We state the optimal switch price in the following theorem.

THEOREM 3.1: For a given bargained price of p_O, the optimal switch price that maximizes the buyer's profit is given by

$$p_{sw}^*(p_O) = p_O - c_p.$$

The buyer offers optimal switch price $p_{sw}^*(p_O) = p_O - c_p$ to the primary supplier. The optimal switch price satisfies the option participation constraint at equality. Because the switch price equals the supplier's profit of $p_O - c_p$ at bargained price p_O, the supplier's profit is unaffected by the future event of whether the option is exercised or not. The expected profit of the primary supplier in Phase 2 and Phase 3 is given by

$$u_O(\cdot; p_O) = p_O - c_p. \tag{7}$$

Next, we analyze the second auction with a reserve price. We assume that the buyer (or the procurement system) prevents the primary supplier from participating in the second auction. This simplifies the auction process because the primary supplier will not submit a bid even though he is allowed to participate: The buyer will set the reserve price less than $p_O - p_{sw}$ because she wants to pay less by opening the second auction (it is not profitable for the primary supplier to be a winner of the auction). We consider the private value model where the bidders' opportunity costs are independent and identically distributed. Therefore, the results in Section 2.1 can be used to analyze the second auction.

If the buyer opens the second auction with reserve price r, her payment is the auction price plus the switch price if the option is exercised (i.e., there is a winner in the second auction) and the bargained price otherwise. The probability of having a winning bid is $G_n(r)$. The expected profit of the buyer in Phase 3 is

$$\begin{aligned}
&\pi_O(r; p_O)\\
&\quad = v - G_n(r) \cdot (E[\tilde{P}_A(r)] + p_{sw}^*(p_O)) - \bar{G}_n(r) \cdot p_O\\
&\quad = v - G_n(r) \cdot (E[\tilde{P}_A(r)] + p_O - c_p) - \bar{G}_n(r) \cdot p_O\\
&\quad = v + G_n(r) \cdot (c_p - E[\tilde{P}_A(r)]) - p_O\\
&\quad = v + H(r) - p_O,
\end{aligned} \tag{8}$$

where we define

$$H(r) \equiv G_n(r) \cdot (c_p - E[\tilde{P}_A(r)]). \tag{9}$$

The optimal reserve price for the second auction, denoted by r_O^*, is the price r that maximizes $\pi_O(r; p_O)$. Maximizing $\pi_O(r; p_O)$ is equivalent to maximizing $H(r)$, which corresponds to $\Pi_A(r)$ in Eq. (1) if v is replaced by c_p. Therefore, if we apply c_p in the place of v to Eq. (2) for r^*, we can acquire Eq. (10) for r_O^* as in Theorem 3.2.

THEOREM 3.2: Under the option mechanism, the optimal reserve price for the second auction, denoted by r_O^*, is the solution r of the equation

$$c_p - r = \frac{F(r)}{f(r)}. \tag{10}$$

The optimal expected profit of the buyer in Phase 3 is

$$\pi_O(r_O^*; p_O) = v + H(r_O^*) - p_O, \tag{11}$$

where $H(r_O^*) > 0$.

PROOF: We explained how Eq. (10) for r_O^* is obtained earlier. Eq. (11) can be obtained by applying r_O^* to Eq. (18) for $\pi_O(r; p_O)$. Thus, we just need to prove $H(r_O^*) > 0$ or equivalently $c_p > E[\tilde{P}_A(r_O^*)]$. We have

$$\tilde{P}_A(r_O^*) \leq r_O^* = c_p - \frac{F(r_O^*)}{f(r_O^*)} < c_p.$$

The first inequality follows because the payment of the first-price or the second-price auctions is less than or equal to the prespecified reserve price. The equality comes from Eq. (10). □

Here is the meaning of Eq. (10) for r_O^*. The buyer can pay $p_{sw}^*(p_O)$ instead of p_O to the primary supplier when there is a winner in the second auction, so the implicit valuation from the second auction is the difference of the two prices, which is $p_O - p_{sw}^*(p_O) = p_O - p_O + c_p = c_p$. Therefore, we can compute the optimal reserve price for the second auction by applying the implicit valuation ($= c_p$) to Eq. (2) for r^*. It is interesting to know that reserve price r_O^* does not depend on bargained price p_O since p_O is cancelled out in the implicit value calculation. If the buyer can switch suppliers (i.e., the option is exercised), the buyer can pay the lower price $\tilde{P}_A(r_O^*)$ instead of p_O to purchase the changed products or services. In this case, the switch benefit of $p_O - \tilde{P}_A(r_O^*)$ is created. From the switch benefit, the buyer pays the switch price $p_{sw}^*(p_O)$ to the primary supplier, leaving $c_p - \tilde{P}_A(r_O^*)$ as her net benefit. The switch benefit, buyer's net switch benefit, and the optimal switch price are shown in Fig. 2. The net switch benefit of the buyer is $c_p - \tilde{P}_A(r_O^*)$ if the option is exercised and zero otherwise, which means that the expected net switch benefit is $H(r_O^*)$. We define $\tilde{H}(r_O^*) = c_p - \tilde{P}_A(r_O^*)$. Then we have $H(r_O^*) = E[\tilde{H}(r_O^*) \cdot I\{Y_n \leq r_O^*\}]$. Therefore, the expected profit of the buyer by opening the optimal second auction is her expected profit under the bargaining process ($= v - p_O$) plus her expected net switch benefit ($= H(r_O^*)$), leading to Eq. (11) for $\pi_O(r_O^*; p_O)$.

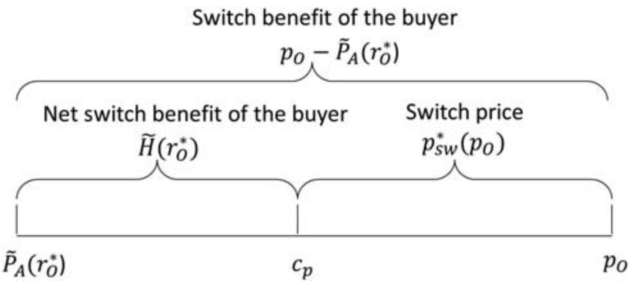

Figure 2. Buyer's switch benefit when the option is exercised.

3.2. Phase 1: Bargaining

Now we analyze the bargaining process in Phase 1. If bargaining fails, the buyer pays penalty cost K to the primary supplier and opens a new auction. The optimal expected profit the buyer obtains from the new auction is Π_A^* in Eq. (3). If bargaining is successful, the option mechanism is used with the bargained price, denoted by p_O. Under the option mechanism, the expected profit of the buyer and the primary supplier are given by Eqs. (11) and (7), respectively. Note that $H(r_O^*)$ does not depend on p_O (r_O^* does not depend on p_O). Hence, the expected profit of the buyer is decreasing in p_O and that of the primary supplier is increasing in p_O. Table 2 summarizes the payoffs of the buyer and the primary supplier depending on the result of bargaining. If the sum of failure payoffs is greater than the sum of success payoffs (i.e., $\Pi_A^* > v + H(r_O^*) - c_p$), no bargained price exists. However, if $\Pi_A^* \le v + H(r_O^*) - c_p$, the bargained price is determined using the bargaining powers of the two parties. As the bargaining power of the buyer is λ and that of the primary supplier is $1-\lambda$, the outcome of the contract values for the buyer is $\lambda \cdot (v + H(r_O^*) - \Pi_A^* - c_p)$ and that for the primary supplier is $(1-\lambda)\cdot(v + H(r_O^*) - \Pi_A^* - c_p)$. The bargained price is determined such that the ratio between $v + H(r_O^*) - p_O + K - \Pi_A^*$ and $p_O - c_p - K$ is $\frac{\lambda}{1-\lambda}$. We prove that it is always true that $v + H(r_O^*) - \Pi_A^* - c_p \ge 0$ and state the bargained price in the following theorem

THEOREM 3.3: In the option mechanism,

$$v + H(r_O^*) - \Pi_A^* - c_p \ge 0. \tag{12}$$

Therefore, under the option mechanism, there always exists the bargained price, which is given by

$$p_O = \lambda \cdot c_p + (1-\lambda)\cdot(\bar{G}_n(r^*)\cdot v + E[P_A(r^*)] + H(r_O^*)) + K. \tag{13}$$

PROOF: The inequality in Eq. (12) holds because

$$\begin{aligned} &v + H(r_O^*) - \Pi_A^* - c_p \\ &= H(r_O^*) + \bar{G}_n(r^*)v + E[P_A(r^*)] - c_p \\ &\ge H(r_O^*) + \bar{G}_n(r^*)c_p + E[P_A(r^*)] - c_p \\ &= H(r_O^*) - (G_n(r^*)c_p - E[P_A(r^*)]) \\ &= H(r_O^*) - H(r^*) \\ &\ge 0. \end{aligned}$$

The first inequality follows because $v \ge c_p$. The second inequality follows because r_O^* is the solution r maximizing $H(r)$. □

Table 2. Payoffs depending on bargaining outcome under option mechanism

Outcome	Buyer's payoff	Primary supplier's payoff	Sum
Failure	$\Pi_A^* - K$	K	Π_A^*
Success	$v + H(r_O^*) - p_O$	$p_O - c_p$	$v + H(r_O^*) - c_p$
Value (Diff.)	$v + H(r_O^*) - p_O + K - \Pi_A^*$	$p_O - c_p - K$	$v + H(r_O^*) - \Pi_A^* - c_p$

Bargained price p_O in Eq. (13) is greater than penalty cost K and linearly increasing in K. We can also consider that the bargained price includes the penalty cost similar to p_Q in Eq. (4) for the sequential BA. Optimal switch price $p_{sw}^*(p_O)$ satisfies

$$\begin{aligned} p_{sw}^*(p_O) &= p_O - c_p \\ &= (1-\lambda)\cdot(\bar{G}_n(r^*)\cdot v + E[P_A(r^*)] \\ &\quad + H(r_O^*) - c_p) + K \\ &= (1-\lambda)\cdot(v + H(r_O^*) - \Pi_A^* - c_p) + K \\ &\ge K. \end{aligned} \tag{14}$$

The last inequality is from Eq. (12). Thus, switch price $p_{sw}^*(p_O)$ is also greater than K and linearly increasing in K.

Under the option mechanism, the expected profit of the buyer is

$$\Pi_O = \lambda \cdot (v + H(r_O^*) - c_p) + (1-\lambda)\cdot \Pi_A^* - K, \tag{15}$$

and the expected profit of the primary supplier is

$$U_O = (1-\lambda)\cdot(v + H(r_O^*) - c_p - \Pi_A^*) + K. \tag{16}$$

Note that the expected profit of the primary supplier is equal to the optimal switch price of $p_{sw}^*(p_O)$.

4. COMPARISON

In this section, we compare the reserve prices, the bargained prices, and the expected profits under the sequential BA and the option mechanism.

4.1. Comparison of Reserve Prices

We compare the reserve prices. Under the sequential BA, if bargaining in phase 1 has failed, the optimal reserve price for the new auction is r^* in Eq. (2). Under the option mechanism, the optimal reserve price for the second auction is r_O^* in Eq. (10). In Theorem 4.1, we show that the optimal reserve price of the option mechanism, r_O^*, is less than or equal to the reserve price r^* of the sequential BA. This means that the buyer is more aggressive in setting the reserve price under the option mechanism. This is intuitively obvious because the buyer can make the contract with the primary supplier when there is no winner in the second auction. Furthermore, in the second auction the buyer attempts to find the lowest price which is less than c_p (i.e., c_p is the reference price for the buyer), whereas in the new auction she tries to find the lowest price without any reference price.

THEOREM 4.1: The optimal reserve price under the option mechanism ($= r_O^*$) is less than or equal to the reserve price under the sequential BA ($= r^*$), $r_O^* \leq r^*$.

PROOF: We have $v = r^* + \frac{F(r^*)}{f(r^*)}$ and $c_p = r_O^* + \frac{F(r_O^*)}{f(r_O^*)}$ by Eqs. (4) and (10). Because $v \geq c_p$ and $r + \frac{F(r)}{f(r)}$ is increasing in r, we have $r_O^* \leq r^*$. □

4.2. Comparison of Bargained Prices

We compare the bargained price under the sequential BA ($= p_Q$) with the bargained price under the option mechanism ($= p_O$). p_Q is defined in Eq. (4) and p_O is defined in Eq. (13). By comparing the two equations, we can easily have that if $\Pi_A^* \leq v - c_p$,

$$p_O = p_Q + (1 - \lambda) \cdot H(r_O^*). \tag{17}$$

Because $H(r_O^*) > 0$ by Theorem 3.2, p_O is higher than p_Q by $(1 - \lambda) \cdot H(r_O^*)$, which is the multiplication of the bargaining power of the primary supplier and the expected net switch benefit of the buyer in the second auction. The primary supplier can increase the bargained price using his bargaining power of $(1-\lambda)$ from the buyer's expected net switch benefit in the second auction. This means that the primary supplier is able to match the potential benefit that the buyer gets from the option mechanism (i.e., $H(r_O^*)$) through bargaining with the buyer using his own bargaining power. If his bargaining power is negligible (i.e., $1 - \lambda = 0$), then the bargained price remains same (i.e., $p_O = p_Q$).

It is also interesting to know that bargained price p_Q exists only when the cost of the primary supplier is not greater than $v - \Pi_A^*$. If the cost of the primary supplier is high enough ($c_p > v - \Pi_A^*$), the buyer does not make a contract with the primary supplier and instead opens the new auction under the sequential BA. However, bargained price p_O always exists whatever the cost of the primary supplier is. This means that even though the cost of the primary supplier is high, the buyer makes a contract with the primary supplier at the bargained price ($= p_O$) and tries to find another supplier in the second auction while keeping the primary supplier as another supplier for the case of no auction winner.

4.3. Comparison of Expected Profits

The expected profits under the sequential BA are given by Eqs. (5) and (6). The expected profits under the option mechanism are given by Eqs. (15) and (16). We show that the option mechanism is mutually beneficial for the buyer and the primary supplier compared to the sequential BA. We state this in Theorem 4.2.

THEOREM 4.2: Under the option mechanism, the expected benefit of the buyer over the sequential BA is

$$\Pi_O - \Pi_Q = \begin{cases} \lambda \cdot H(r_O^*) & if\ v - c_p \geq \Pi_A^* \\ \lambda \cdot (v + H(r_O^*) - \Pi_A^* - c_p) & if\ v - c_p < \Pi_A^*. \end{cases}$$

The expected benefit of the primary supplier over the sequential BA is

$$U_O - U_Q = \begin{cases} (1-\lambda) \cdot H(r_O^*) & if\ v - c_p \geq \Pi_A^* \\ (1-\lambda) \cdot (v + H(r_O^*) - \Pi_A^* - c_p) & if\ v - c_p < \Pi_A^*. \end{cases}$$

Note that $H(r_O^*) > 0$ by Theorem 3.2 and $v + H(r_O^*) - \Pi_A^* - c_p \geq 0$ by Theorem 3.3. Hence, both the buyer and the primary supplier benefit from the option mechanism when compared to the sequential BA.

The sequential BA can be thought of as a special case of the option mechanism where the switch price is set so low as to result in no agreed option. For the option to be accepted, both the buyer and the primary supplier should see some benefit (otherwise, they would not have entered into the option phase). Essentially, the additional benefit of the option mechanism comes from the other bidders (in the second auction), and this benefit is split between the buyer and the primary supplier in the bargaining phase. Furthermore, there is an indirect cost that is not formally included in the sequential BA. It is associated with not finding any supplier at all. The case happens when there is no auction winner in the new auction after bargaining fails. This indirect cost is not incurred in the option mechanism, which makes it more desirable for the buyer.

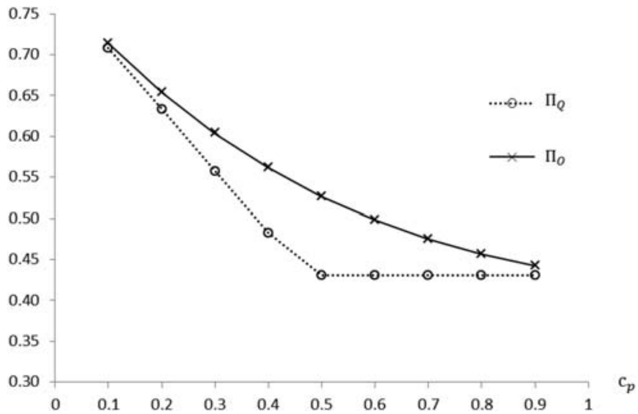

Figure 3. Comparison of buyer profits between the sequential BA and the option mechanism.

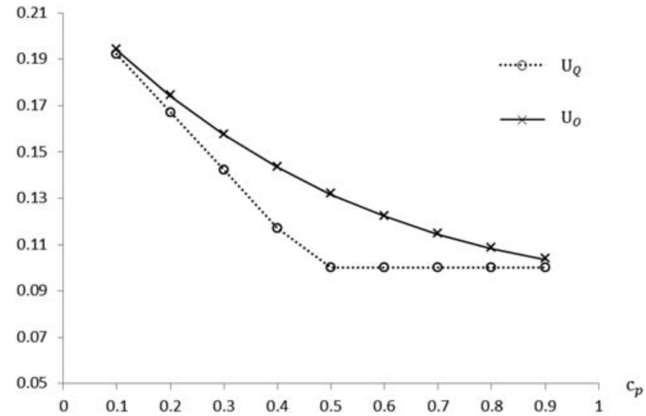

Figure 4. Comparison of primary supplier profits between the sequential BA and the option mechanism.

Note that $H(r_O^*)$ is increasing in c_p because

$$\begin{aligned}\frac{\partial H(r_O^*)}{\partial c_p} &= G_n(r_O^*) + \frac{\partial H(r_O^*)}{\partial r_O^*} \cdot \frac{\partial r_O^*}{\partial c_p} \\ &= G_n(r_O^*) \\ &> 0.\end{aligned}$$

The last equality follows because r_O^* is the first-order solution maximizing $H(r_O^*)$. However, $v + H(r_O^*) - \Pi_A^* - c_p$ is decreasing in c_p because

$$\begin{aligned}\frac{\partial(v + H(r_O^*) - \Pi_A^* - c_p)}{\partial c_p} &= G_n(r_O^*) - 1 \\ &< 0.\end{aligned}$$

Therefore, the expected benefit over the sequential BA in Theorem 4.2 is increasing in the cost of the primary supplier when the cost is less than $v - \Pi_A^*$ and is decreasing when the cost is greater than $v - \Pi_A^*$. In other words, the option mechanism is especially effective when the cost of the primary supplier is neither too high nor too low in the cost range found by the auction market. Under this condition, the buyer has greatest uncertainty about whether or not the cost of the primary supplier is higher or lower than the other suppliers in the auction. The greatest benefit of the option mechanism over the sequential BA is achieved when $c_p = v - \Pi_A^*$. We visualize the mutual benefit in the following example.

EXAMPLE 4.1: Assume that the valuation of the changed terms for the buyer is $v = 1$. We change the cost of the primary supplier, c_p, from 0.1 to 0.9 incrementing by 0.1. In the auction, it is assumed that there are 3 bidders (i.e., $n = 3$) and $C_i \sim$ uniform[0,1] (uniformly distributed opportunity costs). The cost of breaking the contract with the primary supplier is assumed to be 0.1 ($K = 0.1$). The bargaining power of the buyer is given by $\lambda = 0.75$. We calculate the expected profits of the buyer and the primary supplier for all $c_p = 0.1, \ldots, 0.9$ under the two alternatives and draw the scatter plots in Figs. 3 and 4, respectively. The horizontal axis represents c_p. For all the plots, the circular marks are for the sequential BA and the cross marks are for the option mechanism. In Fig. 3, we can see that the expected profit of the buyer under the optimal option mechanism is greater than that under the sequential BA for all c_p values (i.e., $\Pi_O > \Pi_Q$). In Fig. 4, we can also see that the expected profit of the primary supplier under the optimal option mechanism is greater than that under the sequential BA (i.e., $U_O < U_Q$).

5. VARIANT ON OPTION MECHANISM: BARGAINING ON SWITCH PRICE

The option mechanism in this article uses the ultimatum game to determine the switch price. In this section, we consider the case where the switch price in Phase 2 of the option mechanism is bargained with relative bargaining powers. To make it more general, we denote the bargaining power of the buyer by $\hat{\lambda} \in [0, 1]$ so that it can be different from the bargaining power λ in Phase 1 of the option mechanism. We show (1) the ultimatum game (i.e., the buyer has all the bargaining power, $\hat{\lambda} = 1$) is the optimal choice for both the buyer and the primary supplier and (2) the switch price determined in the ultimatum game results in the solution coordinating the supply chain consisting of the two parties. Thus, if the bargaining power for the switch price is also a parameter that the two parties can agree on, they have the optimal result by using the ultimatum game studied in Section 3. We first analyze the second auction in Phase 3 and then study the bargaining process to determine the switch price in Phase 2. In the last step, we study the bargaining process in Phase 1. All the proofs for this section can be found in Appendix B.

Table 3. Payoffs depending on bargaining outcome on switch price (Phase 2)

Bargaining outcome	Buyer's payoff	Primary supplier's payoff	Sum
Failure	$v - \hat{p}_O$	$\hat{p}_O - c_p$	$v - c_p$
Success	$\hat{\pi}_O(\hat{r}_O^*; \hat{p}_{sw}, \hat{p}_O)$	$\hat{u}_O(\hat{r}_O^*; \hat{p}_{sw}, \hat{p}_O)$	$v + H(\hat{r}_O^*) - c_p$
Contract value (Diff.)	$\hat{\pi}_O(\hat{r}_O^*; \hat{p}_{sw}, \hat{p}_O) - v + \hat{p}_O$	$\hat{u}_O(\hat{r}_O^*; \hat{p}_{sw}, \hat{p}_O) - \hat{p}_O + c_p$	$H(\hat{r}_O^*)$

5.1. Phase 3: Second Auction

First, we analyze the second auction (Phase 3) while assuming the bargained price $\hat{p}_O$ and the switch price $\hat{p}_{sw}$ are given. Let the reserve price be r. The buyer pays the auction price plus the switch price if there is a winner in the second auction. Otherwise, she pays the bargained price. The probability of having a winning bid is $G_n(r)$. The expected profit of the buyer in Phase 3 is

$$\begin{aligned}\hat{\pi}_O(r; \hat{p}_{sw}, \hat{p}_O) &= v - G_n(r) \cdot (E[\tilde{P}_A(r)] + \hat{p}_{sw}) - \bar{G}_n(r) \cdot \hat{p}_O \\ &= v + G_n(r) \cdot (\hat{p}_O - \hat{p}_{sw} - E[\tilde{P}_A(r)]) - \hat{p}_O. \quad (18)\end{aligned}$$

The corresponding expected profit of the primary supplier is

$$\begin{aligned}\hat{u}_O(r; \hat{p}_{sw}, \hat{p}_O) &= G_n(r) \cdot \hat{p}_{sw} + \bar{G}_n(r) \cdot (\hat{p}_O - c_p) \\ &= G_n(r) \cdot (\hat{p}_{sw} - \hat{p}_O + c_p) + \hat{p}_O - c_p. \quad (19)\end{aligned}$$

The optimal reserve price for the buyer is the price r maximizing $\hat{\pi}_O(r; \hat{p}_{sw}, \hat{p}_O)$ in Eq. (18), or equivalently the solution r maximizing $G_n(r) \cdot (\hat{p}_O - \hat{p}_{sw} - E[\tilde{P}_A(r)])$. We let the optimal reserve price be $\hat{r}_O^* = \hat{r}_O^*(\hat{p}_{sw}, \hat{p}_O)$ for notational simplicity. We state the result in the following theorem.

THEOREM 5.1: Under the bargaining game for the switch price, the optimal reserve price for the second auction in Phase 3, denoted by $\hat{r}_O^*$, is the solution r of the equation

$$\hat{p}_O - \hat{p}_{sw} - r = \frac{F(r)}{f(r)}, \quad (20)$$

where the bargained price in Phase 1 and the switch price in Phase 2 are given by $\hat{p}_O$ and $\hat{p}_{sw}$ respectively.

5.2. Phase 2: Bargaining for Switch Price

Now we consider Phase 2 of the bargaining game for the switch price. If bargaining fails, the buyer makes a contract with the primary supplier at bargained price $\hat{p}_O$. If the switch price is agreed on $\hat{p}_{sw}$, the buyer opens a second auction with reserve price $\hat{r}_O^*$ given in Eq. (20). The payoffs of the two parties depending on the result of bargaining are summarized in Table 3. If $H(\hat{r}_O^*) \geq 0$, the switch price is determined such that the ratio between $\hat{\pi}_O(\hat{r}_O^*; \hat{p}_{sw}, \hat{p}_O) - v + \hat{P}_O$ and $\hat{u}_O(\hat{r}_O^*; \hat{p}_{sw}, \hat{p}_O) - \hat{P}_O + c_p$ is $\frac{\hat{\lambda}}{1-\hat{\lambda}}$. We prove that the switch price is always bargained (i.e., it is always true that $H(\hat{r}_O^*) \geq 0$) and state the bargained switch price in the following theorem.

THEOREM 5.2: Under the bargaining game for the switch price, the switch price is always determined through bargaining because $H(\hat{r}_O^*) \geq 0$. The bargained switch price is

$$\hat{p}_{sw} = \hat{p}_O - (1 - \hat{\lambda}) \cdot E[\tilde{P}_A(\hat{r}_O^*)] - \hat{\lambda} \cdot c_p. \quad (21)$$

In addition, the optimal reserve price for the second auction is maximized when the buyer has all the bargaining power in Phase 2 (i.e., $\hat{\lambda} = 1$).

The optimal reserve price for the second auction depends on bargained switch price $\hat{p}_{sw}$ in Phase 2 and again depends on bargaining power $\hat{\lambda}$. In Theorem 5.2, we show that the optimal reserve price is maximized when $\hat{\lambda} = 1$. Therefore, the buyer sets the highest reserve price when she has all the bargaining power in Phase 2. We discuss the implication of this when we compare the expected profits in Section 5.4. Applying the switch price in Eq. (21) to the expected profits in Eqs. (18) and (19), we have the expected profit of the buyer given by

$$\hat{\pi}_O(\hat{p}_O) = v - \hat{p}_O + \hat{\lambda} \cdot H(\hat{r}_O^*)$$

and the expected profit of the primary supplier given by

$$\hat{u}_O(\hat{p}_O) = \hat{p}_O - c_p + (1 - \hat{\lambda}) \cdot H(\hat{r}_O^*).$$

The function of $H(r)$ is defined in Eq. (9).

5.3. Phase 1: Bargaining

Finally, we consider Phase 1 of bargaining. Table 4 summarizes the payoffs in this phase. The bargained price is determined if $v + H(\hat{r}_O^*) - \Pi_A^* - c_p \geq 0$, and bargaining terminates otherwise. If $v + H(\hat{r}_O^*) - \Pi_A^* - c_p \geq 0$, bargained price $\hat{p}_O$ is determined such that

$$\hat{p}_O = \lambda \cdot c_p + (1 - \lambda) \cdot (v - \Pi_A^*) + (\hat{\lambda} - \lambda) \cdot H(\hat{r}_O^*) + K. \quad (22)$$

Table 4. Payoffs depending on bargaining outcome in Phase 1

Bargaining outcome	Buyer's payoff	Primary supplier's payoff	Sum
Failure	$\Pi_A^* - K$	K	Π_A^*
Success	$\hat{\pi}_O(\hat{p}_O)$	$\hat{u}_O(\hat{p}_O)$	$v + H(\hat{r}_O^*) - c_p$
Contract value (Diff.)	$\hat{\pi}_O(\hat{p}_O) - \Pi_A^* + K$	$\hat{u}_O(\hat{p}_O) - K$	$v + H(\hat{r}_O^*) - c_p - \Pi_A^*$

Therefore, the expected profit of the buyer is

$$\hat{\Pi}_O = \begin{cases} \Pi_A^* - K + \lambda \cdot (v + H(\hat{r}_O^*) - \Pi_A^* - c_p) \\ \qquad\qquad \text{if } v + H(\hat{r}_O^*) - \Pi_A^* - c_p \geq 0 \\ \Pi_A^* - K \qquad \text{otherwise} \end{cases}$$

and the expected profit of the primary supplier is

$$\hat{U}_O = \begin{cases} K + (1-\lambda) \cdot (v + H(\hat{r}_O^*) - \Pi_A^* - c_p) \\ \qquad\qquad \text{if } v + H(\hat{r}_O^*) - \Pi_A^* - c_p \geq 0 \\ K \qquad \text{otherwise.} \end{cases}$$

5.4. Comparison of Expected Profits

We compare the expected profits of the buyer and the primary supplier with Π_O [Eq. (20)] and U_O [Eq. (16)] in Section 3. We have

$$\Pi_O - \hat{\Pi}_O = \begin{cases} \lambda \cdot (H(r_O^*) - H(\hat{r}_O^*)) \\ \qquad\qquad \text{if } v + H(\hat{r}_O^*) - \Pi_A^* - c_p \geq 0 \\ \lambda \cdot (v + H(r_O^*) - \Pi_A^* - c_p) \quad \text{otherwise} \end{cases}$$

and

$$U_O - \hat{U}_O = \begin{cases} (1-\lambda) \cdot (H(r_O^*) - H(\hat{r}_O^*)) \\ \qquad\qquad \text{if } v + H(\hat{r}_O^*) - \Pi_A^* - c_p \geq 0 \\ (1-\lambda) \cdot (v + H(r_O^*) - \Pi_A^* - c_p) \text{ otherwise.} \end{cases}$$

Because $H(r)$ is maximized at $r = r_O^*$ and $v + H(r_O^*) - \Pi_A^* - c_p \geq 0$ by Theorem 3.3, we have $\Pi_O \geq \hat{\Pi}_O$ and $U_O \geq \hat{U}_O$. Therefore, both the buyer and the primary supplier can maximize their profits under the ultimatum game where the buyer makes a take-it-or-leave-it offer for the switch price. The value of $H(\hat{r}_O^*)$ is the switch benefit of the option mechanism and is split between the buyer and the primary supplier in Phase 1. In Theorem 5.2, we show that $\hat{r}_O^*$ is maximized when $\hat{\lambda} = 1$. This means that the buyer sets a lower reserve price for the second auction, resulting in less surplus for the two parties if $\hat{\lambda} < 1$. It is interesting to note that the primary supplier can maximize his profit by abandoning all of his bargaining power in Phase 2 under the option mechanism.

5.5. Two Extreme Values of Bargaining Power in Phase 2

We consider two extreme values of bargaining power of the buyer in Phase 2: $\hat{\lambda}$=1 and $\hat{\lambda}$=0. First, if the buyer has all the bargaining power (i.e., $\hat{\lambda} = 1$), it is the proposed option mechanism analyzed in Section 3 (i.e., the ultimatum game where the buyer makes a take-it-or-leave-it offer).

Next, we consider the case where the primary supplier has all the bargaining power (i.e., $\hat{\lambda} = 0$). In this case, the switch price proposed by the primary supplier is simply the bargained price determined in Phase 1. The response of the buyer to the switch price is to make the reserve price equal to zero. This means that the option is not exercised because there is no auction winner with reserve price equal to zero. In other words, the buyer makes the second auction default (so that the option is not exercised) to respond such a high switch price. Or the buyer can simply reject the switch price. The bargained price in Phase 1 is exactly the same as the bargained price in the sequential BA (= p_Q) in Eq. (4). Hence, if the primary supplier has all the bargaining power (or the ultimatum game where the primary supplier makes a take-it-or-leave-it offer), the option mechanism is reduced to the sequential BA. We show all this in Theorem 5.3.

THEOREM 5.3: If the primary supplier has all the bargaining power in Phase 2 (i.e., $\hat{\lambda} = 0$), the option mechanism is simply the sequential BA. The optimal reserve price for the second auction is $\hat{r}_O^* = 0$ and the switch price is equal to the bargained price in Phase 1.

$$\hat{p}_{sw} = \hat{p}_O = p_Q = \lambda \cdot c_p + (1-\lambda) \cdot (v - \Pi_A^*) + K.$$

5.6. Supply Chain Coordination

The benefit of the option mechanism comes from the other bidders in the second auction. Then we may wonder which reserve price maximizes the surplus. We consider the supply chain consisting of the buyer and the primary supplier. The profit of the supply chain in Phase 3 is the sum of Eqs. (18) and (19), which is

$$\hat{\pi}_O(r; \hat{p}_{sw}, \hat{p}_O) + \hat{u}_O(r; \hat{p}_{sw}, \hat{p}_O) = v + H(r) - c_p.$$

The optimal reserve price maximizing the supply chain is r_O^* in Eq. (10). Therefore, the option mechanism with the ultimatum game where the buyer makes a take-it-or-leave-it offer for the switch price (as proposed in the paper, $\hat{\lambda} = 1$) maximizes the surplus and coordinates the supply chain.

6. CONCLUSIONS

Change orders occur frequently after the signing of contracts, especially when the contract is long-term or contractual designs are complex. We propose an efficient means of handling change orders: the option mechanism. Under the option mechanism, the buyer and the primary supplier negotiate a new price for the changed products or services, after which the buyer opens a second auction to find another low-cost supplier while keeping the previously bargained price. The buyer exercises the option when there is a winner in the second auction, and the primary supplier transfers the selling privilege to the auction winner in exchange for a prespecified switch price. We show how the optimal switch price (the price at which the primary supplier accepts the option terms) and the optimal reserve price (for the second auction) are determined under the operation of the option mechanism.

Additionally, we demonstrate that the option mechanism is beneficial, for both the buyer and the primary supplier, compared to the sequential BA, wherein a bargaining phase is followed by an auction phase if bargaining fails. The key benefit of the option mechanism comes from the possibility of finding a lower cost from the other, second-auction bidders, which benefit is split between the buyer and the primary supplier in the bargaining phase. In this way, the split share increases in his (or her) bargaining power. The benefit of the option mechanism over the sequential BA is most significant when the cost of the primary supplier is neither too high nor too low in the cost range determined by the auction market. At this point, the buyer has the greatest uncertainty as to whether the cost of the primary supplier is higher or lower than those of the other suppliers in the auction. The option mechanism also coordinates the supply chain consisting of the two parties, specifically by extracting the maximum surplus, whereas the sequential BA does not.

There are two indirect costs that are not formally included in our model. The first is the cost that the buyer pays when she fails to find a supplier for the changed terms (this cost is significant in the construction industry). In the sequential BA, the buyer may end up with no supplier. This happens when there is no auction winner in the new auction after bargaining has failed in the previous phase. Note that there is always a supplier in the option mechanism (either the primary supplier or an auction winner in the second auction), because there always exists the bargained price in Phase 1, as proved in Theorem 3.3. The second indirect cost is associated with the loss of good-will. If the buyer opens an auction to find a new supplier before attempting to negotiate with the existing primary supplier (i.e., according to the sequential AB as analyzed in Appendix A), she might lose any good-will that she had earned with the primary supplier. The primary supplier, furthermore, might choose to not return to the bargaining table after the auction. Also, it is unclear what the buyer is committing to in the auction; in other words, the winner of the auction might or might not end up supplying the buyer. If the buyer reneges on her commitment to the winner of the auction, her reputation could be negatively affected. Accounting for the two indirect costs (not incurring in the proposed option mechanism) will make the option mechanism more preferable to the sequential BA and AB mechanisms.

In this article, we have considered the case in which a change order occurs only once in the postcontract period. We are considering, for future work, extending the analysis to cases wherein multiple change orders occur after contract signing. Further, as our current analysis does not consider how the primary supplier is initially chosen, we might extend our analytical scope to include the selection of the primary supplier, particularly as this outcome can affect bids in the option mechanism.

Also, we have assumed that the buyer knows the cost of the primary supplier through the relationship that had been built over the course of a long-term contract. However, if the change order contains some factors that are difficult to estimate, the buyer, in the bargaining stage, will have only incomplete knowledge of the cost of the primary supplier. For example, whereas a print shop can estimate the cost of the primary supplier when the change order includes only a quantity adjustment, it might not be able to estimate the cost when the change order includes a delivery time change. We will leave this issue of incomplete knowledge of cost to future research. In one-sided uncertainty, unlike the case of perfect information, there are many notions of equilibrium (e.g., signalling equilibrium in Admati and Perry [1] and Cramton [5], screening equilibrium in Grossman and Perry [11] and Gul and Sonnenschein [12]); that which is the best predictor of bargainers' behavior is what needs to be determined.

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Table 5. Payoffs under sequential AB with an auction winner in Phase 2

Bargaining outcome	Buyer's payoff	Primary supplier's payoff	Sum
Failure	$v - \tilde{P}_A(r) - K$	K	$v - \tilde{P}_A(r)$
Success	$v - p_{AB}$	$p_{AB} - c_p$	$v - c_p$
Contract value (Diff.)	$\tilde{P}_A(r) - p_{AB} + K$	$p_{AB} - c_p - K$	$\tilde{P}_A(r) - c_p$

Table 6. Payoffs under sequential AB with no auction winner in Phase 2

Bargaining outcome	Buyer's payoff	Primary supplier's payoff	Sum
Failure	$-K$	K	0
Success	$v - p_{AB}$	$p_{AB} - c_p$	$v - c_p$
Contract value (Diff.)	$v - p_{AB} + K$	$p_{AB} - c_p - K$	$v - c_p$

APPENDIX A: SEQUENTIAL AUCTION-BARGAINING

In response to a change order, the buyer may open an auction to find a new supplier before attempting to negotiate with the primary supplier, which we call the sequential auction-bargaining (AB). Here, we analyze the sequential AB and compare it with the option mechanism. We show that the option mechanism is beneficial for the buyer compared to the sequential AB. Thus, there is no incentive for the buyer to open an auction in advance of the negotiation if the option mechanism is available. In addition, the sequential AB does not coordinate the supply chain consisting of the buyer and the primary supplier, whereas the option mechanism does. The benefit of the option mechanism for the buyer comes when the realized auction price turns out to be extremely high in the sequential AB. In this case, the buyer is not able to negotiate a good price. In an extreme case, the buyer may have no auction winner (because the auction market turns out to be extremely unfavorable). She then needs to pay a higher bargained price because she has no outside alternative. However, under the option mechanism, the buyer bargains a price with the primary supplier still with the threat of a potential new auction.

Phase 2: Bargaining

We first consider Phase 2 of bargaining. Let the bargained price be p_{AB}. There are two cases for reserve price r in Phase 1: There is an auction winner and there is no auction winner. We first consider the case where there is an auction winner. Let $\tilde{P}_A(r)$ be the realized auction price. The payoffs of the buyer and the primary supplier are in Table 5 when there is an auction winner. If $\tilde{P}_A(r) \geq c_p$, the bargained price is determined as follows.

$$p_{AB}|\text{winner} = \lambda \cdot c_p + (1-\lambda) \cdot \tilde{P}_A(r) + K.$$

If $\tilde{P}_A(r) < c_p$, the negotiation terminates. The buyer makes a contract with the auction winner and pays penalty cost K to the primary supplier (i.e., payment is $\tilde{P}_A(r) + K$). Therefore, if there is an auction winner, the payment of the buyer is

$$\lambda \cdot \min\left\{\tilde{P}_A(r), c_p\right\} + (1-\lambda) \cdot \tilde{P}_A(r) + K.$$

Next we consider the case where there is no auction winner. Table 6 describes the payoffs of the two parties. Since $v \geq c_p$, the bargained price is always determined at

$$p_{AB}|\text{nowinner} = \lambda \cdot c_p + (1-\lambda) \cdot v + K.$$

Phase 1: Auction

We consider the expected profit of the buyer in Phase 1 of auction. We consider the second-price auction for the easy of analysis. Each bidder receives the auction price when he is the lowest bidder and the negotiation fails in Phase 2. Because the payment function does not depend on his bid, his bidding strategy is to bid truthfully (i.e., to bid his true cost). If there is an auction winner, the buyer pays $\lambda \cdot \min\left\{\tilde{P}_A(r), c_p\right\} + (1-\lambda) \cdot \tilde{P}_A(r) + K$. If there is no auction winner, she pays $\lambda \cdot c_p + (1-\lambda) \cdot v + K$. The expected profit of the buyer is

$$\begin{aligned}\pi_{AB}(r) &= v - G_n(r) \cdot (\lambda \cdot E \min\left\{\tilde{P}_A(r), c_p\right\} + (1-\lambda) \cdot E[\tilde{P}_A(r)] + K) \\ &\quad - \bar{G}_n(r) \cdot (\lambda \cdot c_p + (1-\lambda) \cdot v + K) \\ &= \lambda \cdot G_n(r) \cdot (c_p - E[\min\left\{\tilde{P}_A(r), c_p\right\}]) \\ &\quad + (1-\lambda) \cdot G_n(r) \cdot (v - E[\tilde{P}_A(r)]) + \lambda \cdot (v - c_p) - K. \end{aligned} \tag{A.1}$$

The expected profit has two different forms depending on whether $r \leq c_p$ or not. We let the expected profit be $\underline{\pi}_{AB}(r)$ if $r \leq c_p$ and $\overline{\pi}_{AB}(r)$ if $r > c_p$.

Case 1: $r \leq c_p$ If $r \leq c_p$, then $\min\left\{\tilde{P}_A(r), c_p\right\} = \tilde{P}_A(r)$. In this case, the expected profit is

$$\begin{aligned}\underline{\pi}_{AB}(r) &= \lambda \cdot G_n(r) \cdot (c_p - E[\tilde{P}_A(r)]) \\ &\quad + (1-\lambda) \cdot G_n(r) \cdot (v - E[\tilde{P}_A(r)]) + \lambda \cdot (v - c_p) - K \\ &= \lambda \cdot H(r) + (1-\lambda) \cdot \Pi_A(r) + \lambda \cdot (v - c_p) - K. \end{aligned} \tag{A.2}$$

In the above equation, $H(r)$ is defined in Equation (9) and $\Pi_A(r)$ is defined in Equation (5).

Case 2: $r > c_p$ For the expected profit under the case of $r > c_p$, we have the following lemma.

LEMMA A.1.: If $r > c_p$,

$$G_n(r) \cdot E[\min\left\{\tilde{P}_A(r), c_p\right\}] = c_p \cdot G_n(r) - H(c_p).$$

PROOF:

$$\begin{aligned} &G_n(r) \cdot E[\min\left\{\tilde{P}_A(r), c_p\right\}] \\ &= n \cdot E[E[\min\{Y_{n-1}, r\}, c_p\} \cdot I\{Y_{n-1} > C_n | C_n] \cdot I\{C_n \leq r\}] \\ &= n \cdot E[E[\min\left\{Y_{n-1}, c_p\right\} \cdot I\{Y_{n-1} > C_n | C_n] \cdot (I\left\{C_n \leq c_p\right\} \\ &\quad + I(c_p < C_n \leq r)] \\ &= E[P_A(c_p)] + n \cdot E[c_p \cdot \bar{G}_{n-1}(C_n) \cdot I(c_p < C_n \leq r)] \\ &= E[P_A(c_p)] + c_p \cdot \int_{c_p}^{r} n \cdot \bar{G}_{n-1}(x) \cdot f(x) dx \\ &= E[P_A(c_p)] + c_p \cdot (G_n(r) - G_n(c_p)) \\ &= c_p \cdot G_n(r) - H(c_p). \end{aligned}$$

The first equality follows because $r > c_p$. The third equality follows because $n \cdot E[E[\min\{Y_{n-1}, c_p\} \cdot I\{Y_{n-1} > C_n\}|C_n] \cdot I\{C_n \le c_p\}]$ is the expected auction payment when the reserve price is c_p, which is $E[P_A(c_p)]$. The fifth equality follows because $\frac{dG_n(x)}{dx} = n \cdot \bar{G}_{n-1}(x) \cdot f(x)$. □

Using Lemma A.1, we obtain the expected profit

$$\begin{aligned}\overline{\pi}_{AB}(r) &= \lambda \cdot H(c_p) + (1-\lambda) \cdot G_n(r) \cdot (v - E[\tilde{P}_A(r)]) \\ &\quad + \lambda \cdot (v - c_p) - K \\ &= \lambda \cdot H(c_p) + (1-\lambda) \cdot \Pi_A(r) + \lambda \cdot (v - c_p) - K. \qquad \text{(A.3)}\end{aligned}$$

The following lemma describes the optimal reserve price, denoted by r^*_{AB}, for the sequential AB.

LEMMA A.2: Let $\underline{r}^* = \min\{r_\lambda, c_p\}$ and $\bar{r}^* = \max\{r^*, c_p\}$, where r_λ is the solution r of the following equation.

$$\lambda \cdot c_p + (1-\lambda) \cdot v - r = \frac{F(r)}{f(r)}$$

and r^* is defined in Equation (4). The optimal reserve price for the sequential AB is

$$r^*_{AB} = \begin{cases} \bar{r}^* & \text{if } \overline{\pi}_{AB}(\bar{r}^*) \ge \underline{\pi}_{AB}(\underline{r}^*) \\ \underline{r}^* & \text{otherwise.} \end{cases}$$

PROOF: If $r \le c_p$, the expected profit of the buyer is given in Equation (A.2), which can be written as follows.

$$\underline{\pi}_{AB}(r) = G_n(r) \cdot (\lambda \cdot c_p + (1-\lambda) \cdot v - E[\tilde{P}_A(r)]) + \lambda \cdot (v - c_p) - K.$$

The optimal reserve price is $\underline{r}^* = \min\{r_\lambda, c_p\}$ if $r \le c_p$.

If $r > c_p$, the expected profit of the buyer is given in Equation (A.3). The optimal reserve price is $\bar{r}^* = \max\{r^*, c_p\}$ if $r > c_p$.

Therefore, the optimal reserve price is $r^*_{AB} = \bar{r}^*$ if $\overline{\pi}_{AB}(\bar{r}^*) \ge \underline{\pi}_{AB}(\underline{r}^*)$ and $r^*_{AB} = \underline{r}^*$ otherwise. □

Expected Profit of the Primary Supplier

Next, we consider the expected profit of the primary supplier. If there is an auction winner, his payoff is $p_{AB}|\text{winner} - c_p = (1-\lambda) \cdot (\tilde{P}_A(r) - c_p) + K$ if $\tilde{P}_A(r) \ge c_p$ and K if $\tilde{P}_A(r) < c_p$. Therefore, if there is an auction winner, his payoff is

$$(1-\lambda) \cdot \left(\tilde{P}_A(r) - \min\left\{\tilde{P}_A(r), c_p\right\}\right) + K.$$

If there is no auction winner, his payoff is $p_{AB}|\text{nowinner} - c_p = (1-\lambda) \cdot (v - c_p) + K$.

The expected profit of the primary supplier under the sequential AB is given by

$$\begin{aligned}u_{AB}(r) &= (1-\lambda) \cdot G_n(r) \cdot \left(E[\tilde{P}_A(r)] - E\left[\min\left\{\tilde{P}_A(r), c_p\right\}\right]\right) \\ &\quad + (1-\lambda) \cdot \bar{G}_n(r) \cdot (v - c_p) + K.\end{aligned}$$

Similarly, the expected profit has two different forms depending on whether $r \le c_p$ or not. We denote the expected profit of the primary supplier as $\underline{u}_{AB}$ if $r \le c_p$ and as $\bar{u}_{AB}$ if $r > c_p$.

Case 1: $r \le c_p$

If $r \le c_p$, the expected profit of the primary supplier is

$$\begin{aligned}\underline{u}_{AB}(r) &= (1-\lambda) \cdot \bar{G}_n(r) \cdot (v - c_p) + K \\ &= (1-\lambda) \cdot (v - c_p + H(r) - \Pi_A(r)) + K. \qquad \text{(A.4)}\end{aligned}$$

Case 2: $r > c_p$

If $r > c_p$, the expected profit of the primary supplier is

$$\begin{aligned}\bar{u}_{AB}(r) &= (1-\lambda) \cdot (E[P_A(r)] - c_p \cdot G_n(r) \\ &\quad + H(c_p)) + (1-\lambda) \cdot \bar{G}_n(r) \cdot (v - c_p) + K \\ &= (1-\lambda) \cdot (v - c_p + H(c_p) - G_n(r) \cdot v + E[P_A(r)]) + K \\ &= (1-\lambda) \cdot (v - c_p + H(c_p) - \Pi_A(r)) + K. \qquad \text{(A.5)}\end{aligned}$$

In summary, under the sequential AB, the expected profit of the buyer is

$$\Pi_{AB} = \begin{cases} \underline{\pi}_{AB}(r^*_{AB}) & \text{if } r^*_{AB} \le c_p \\ \overline{\pi}_{AB}(r^*_{AB}) & \text{otherwise} \end{cases}$$

and the expected profit of the primary supplier is given by

$$U_{AB} = \begin{cases} \underline{u}_{AB}(r^*_{AB}) & \text{if } r^*_{AB} \le c_p \\ \bar{u}_{AB}(r^*_{AB}) & \text{otherwise.} \end{cases}$$

Comparison with Option Mechanism

If $r^*_{AB} \le c_p$ (i.e., $r^*_{AB} = r_\lambda$), the expected profit of the buyer is

$$\begin{aligned}\Pi_{AB} &= \underline{\pi}_{AB}(r_\lambda) \\ &= \lambda \cdot H(r_\lambda) + (1-\lambda) \cdot \Pi_A(r_\lambda) + \lambda \cdot (v - c_p) - K \\ &\le \lambda \cdot H(r^*_O) + (1-\lambda) \cdot \Pi^*_A + \lambda \cdot (v - c_p) - K \\ &= \Pi_O.\end{aligned}$$

The inequality follows because r^*_O maximizes $H(r)$ and r^* maximizes $\Pi_A(r)$. If $r^*_{AB} > c_p$ (i.e., $r^*_{AB} = r^*$), the expected profit of the buyer is

$$\begin{aligned}\Pi_{AB} &= \overline{\pi}_{AB}(r^*) \\ &= \lambda \cdot H(c_p) + (1-\lambda) \cdot \Pi^*_A + \lambda \cdot (v - c_p) - K \\ &\le \lambda \cdot H(r^*_O) + (1-\lambda) \cdot \Pi^*_A + \lambda \cdot (v - c_p) - K \\ &= \Pi_O.\end{aligned}$$

Therefore, the buyer has more expected profit under the option mechanism than under the sequential AB.

Now we compare the expected profits of the primary supplier under the sequential AB with that under the option mechanism. If $r^*_{AB} > c_p$ (i.e., $r^*_{AB} = r^*$), we can say that the option mechanism dominates the sequential AB for the primary supplier:

$$\begin{aligned}U_{AB} &= \bar{u}_{AB}(r^*) \\ &= (1-\lambda) \cdot (v - c_p + H(c_p) - \Pi^*_A) + K \\ &\le (1-\lambda) \cdot (v - c_p + H(r^*_O) - \Pi^*_A) + K \\ &= U_O.\end{aligned}$$

However, if $r^*_{AB} \le c_p$ (i.e., $r^*_{AB} = r_\lambda$), the expected profit of the primary supplier is

$$\begin{aligned}U_{AB} &= \underline{u}_{AB}(r_\lambda) \\ &= (1-\lambda) \cdot (v - c_p + H(r_\lambda) - \Pi_A(r_\lambda)) + K.\end{aligned}$$

Note that $U_O = (1-\lambda)\cdot(v - c_p + H(r_O^*) - \Pi_A^*) + K$. Because $H(r_O^*) \geq H(r_\lambda)$ but $\Pi_A^* \geq \Pi_A(r_\lambda)$, we are not able to say that one is larger than the other. This can be explained by understanding the structure of the expected profit of the buyer. When the cost of the primary supplier is high enough, the expected profit of the buyer is close to $\underline{\pi}_{AB}(r)$ (in Equation (A.2)). The first component $(= H(r))$ is the expected benefit of switching suppliers, and the second component $(= \Pi_A(r))$ is the expected benefit from an auction. The buyer optimizes the weighted sum of those two benefits. As her bargaining power is increasing (i.e., greater λ), she gives more weight to the switch benefit. In the extreme case of $\lambda = 1$, r_λ goes down to r_O^*. In this case, the expected profit of the primary supplier is $(1-\lambda)\cdot(v - c_p + H(r_O^*) - \Pi_A(r_O^*)) + K$, which is greater than U_O. Thus, the sequential AB may provide the primary supplier with some benefit over the option mechanism resulting from the auction that is not fully optimized by the buyer. However, this benefit diminishes as the bargaining power of the buyer is decreasing (i.e., less λ). In this case, she gives more weight to the auction benefit. In the other extreme case of $\lambda = 0$, r_λ goes up to r^*. The expected profit of the primary supplier is $(1-\lambda)\cdot(v - c_p + H(r^*) - \Pi_A^*)$. Hence, the sequential AB may provide the primary supplier with less share of the switch benefit.

Supply Chain Coordination

Now we consider the supply chain consisting of the buyer and the primary supplier under the sequential AB. If $r \leq c_p$, the supply chain profit is (see Equation (A.2) and (A.4))

$$\underline{\pi}_{AB}(r) + \underline{u}_{AB}(r) = v - c_p + H(r).$$

If $r > c_p$, the supply chain profit is (see Equation (A.3) and (A.5) of the document)

$$\overline{\pi}_{AB}(r) + \bar{u}_{AB}(r) = v - c_p + H(c_p).$$

The optimal reserve price coordinating the supply chain is r_O^* because $r_O^* \leq c_p$ and $H(r_O^*) \geq H(c_p)$. Note that $r_O^* \leq r_\lambda \leq r^*$. Under the sequential AB, the buyer sets the reserve price higher than the price coordinating the supply chain. However, the option mechanism coordinates the supply chain as shown in Section 5.6.

APPENDIX B: PROOFS OF RESULTS IN SECTION 5

Proof of Theorem 5.2

PROOF: We first prove that $H(\hat{r}_O^*) \geq 0$. We prove this by contradiction. By combining Equation (20) and (21), we have the equation that optimal reserve price $\hat{r}_O^*$ should satisfy:

$$E[\tilde{P}_A(\hat{r}_O^*)] + \hat{\lambda}\cdot(c_p - E[\tilde{P}_A(\hat{r}_O^*)]) = \hat{r}_O^* + \frac{F(\hat{r}_O^*)}{f(\hat{r}_O^*)}. \qquad \text{(B.1)}$$

Assume that $H(\hat{r}_O^*) < 0$. Then $H(\hat{r}_O^*) = G_n(\hat{r}_O^*)\cdot(c_p - E[\tilde{P}_A(\hat{r}_O^*)]) < 0$ or $c_p - E[\tilde{P}_A(\hat{r}_O^*)] < 0$. This means that $\hat{r}_O^* - E[\tilde{P}_A(\hat{r}_O^*)] + \frac{F(\hat{r}_O^*)}{f(\hat{r}_O^*)} < 0$ by Equation (B.1), which is not true because $\hat{r}_O^* \geq E[\tilde{P}_A(\hat{r}_O^*)]$ and $\frac{F(\hat{r}_O^*)}{f(\hat{r}_O^*)} > 0$. Hence, $H(\hat{r}_O^*) \geq 0$.

Next, we prove that $\hat{r}_O^*$ is maximized when $\hat{\lambda} = 1$. $H(\hat{r}_O^*) \geq 0$ implies that $c_p \geq E[\tilde{P}_A(\hat{r}_O^*)]$. Thus, the equation on the left in Equation (B.1) is upper bounded by c_p because it is a convex combination of c_p and $E[\tilde{P}_A(\hat{r}_O^*)]$. Note that the value of the equation on the left is equal to c_p when $\hat{\lambda} = 1$. Because $r + \frac{F(r)}{f(r)}$ is increasing in r, the solution of $\hat{r}_O^*$ of Equation (B.1) is maximized when $\hat{\lambda} = 1$. □

Proof of Theorem 5.3

PROOF: We first prove that $\hat{r}_O^* = 0$. If $\hat{\lambda} = 0$, the optimal reserve price for the second auction satisfies (see Equation (B.1))

$$E[\tilde{P}_A(\hat{r}_O^*)] = \hat{r}_O^* + \frac{F(\hat{r}_O^*)}{f(\hat{r}_O^*)}.$$

Note that $E[\tilde{P}_A(r)] \leq r < r + \frac{F(r)}{f(r)}$ if $r > \underline{c}$ ($\underline{c}$ is the minimum value of the cost of bidder i). Therefore, the reserve price that satisfies the above equation is $\hat{r}_O^* = 0$ at which the values on the right and on the left are all zero.

$\hat{\lambda} = 0$ and $\hat{r}_O^* = 0$ imply $\hat{p}_{sw} = \hat{p}_O$ by Equation (21). In this case, the bargained price is given by $\hat{p}_O = \lambda\cdot c_p + (1-\lambda)\cdot(v - \Pi_A^*) + K$ by Equation (22). □

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