Field Implementation of Eco-driving and Eco-signal System

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Abstract: This paper proposed an integrated system between an eco-driving algorithm and an eco-signal control based on vehicle-to-everything (V2X) communication, and evaluated the system’s environmental benefits. The system calculates eco-speeds using vehicle information (e.g., current locations, vehicle speeds, and acceleration profiles) and signal information. In addition, the system controls current signal phase to improve fuel consumptions if a vehicle can pass the intersection by green time extension. We conducted field tests with three scenarios to evaluate the system using dedicated short-range communication (DSRC) devices and an external device that is able to collect vehicle specific information (e.g., speed and fuel consumption) within controller area network (CAN).

1 INTRODUCTION

Excessive greenhouse gas (GHG) emissions have negative effects including global warming (EPA, Accessed 21 Nov 2016). According to US environmental protection agency, transportation sector caused 26 percent of GHGs such as cars, trucks, etc. (EPA, Accessed 22 Nov 2016). Given that the global number of vehicles could reach 2 billion by 2035 (Voelcker, 2014), it is important to develop vehicle technologies (e.g., electric vehicles) and vehicle control strategies (e.g., eco-driving algorithms) to reduce fuel consumption and greenhouse gas emissions.

In terms of vehicle technologies, electric vehicles have been considered an effective way to reduce emissions, but its market penetration rate is still low. Therefore, it is necessary to apply vehicle control methods to minimize unnecessary vehicle idling time and accelerations and to coordinate vehicle speeds along the traffic signal. For this, it needs to allow vehicle acceleration when a vehicle can pass an intersection by accelerating and to avoid an abrupt stop, a full stop, and idling time as much as possible. We can also take advantage of vehicle-to-vehicle and vehicle-to-infrastructure communications (a.k.a., V2X communication) by a dedicated short range communication (DSRC) protocol so that we can receive the traffic signal information or neighboring vehicles’ trajectory information.

Many researchers have proposed various eco-driving algorithms using V2X communication at a signalized intersection to decrease fuel consumptions. The general idea of these research is to recommend speeds calculated by their own algorithms using the current vehicle location and signal phase and timing (SPaT) data, and to inform to the vehicle through V2X communication. Rakha and Kamalanathsharma used an explicit objective function to minimize the total fuel consumption at a signalized intersection (Rakha and Kamalanathsharma, 2011). Nunzio et al. proposed an algorithm that provides a quick sub-optimal solution for the fuel minimization problem (Nunzio et al, 2013). Xia et al. performed a field test using 4G/LTE network and cloud-based server infrastructure at a fixed-time traffic signalized intersection (Xia et al, 2012). Jin et al. proposed a power-based longitudinal control algorithm for a connected eco-driving system (Jin et al, 2016). These research efforts clearly demonstrated potential benefits of eco-driving system. However, their proposed systems allowed that vehicles only receive SPaT data transmitted via communication links from an infrastructure, it means vehicles’ trajectories should be decided considering given traffic signal status. Therefore, it would be a more effective way to minimize fuel consumptions if a vehicle’s trajectory can be decided considering both the vehicle and signal information via two-way communication between the vehicle and an infrastructure.
In this paper, we proposed an eco-speed guidance system including an eco-driving algorithm and traffic signal controls. The system guides eco-speeds based on SPaT, global positioning system (GPS), and vehicle speed information collected via DSRC at a signalized intersection. Moreover, we applied an eco-signal mechanism in the proposed system that extends remaining green time to avoid unnecessary acceleration/idling time and reduce travel-time of vehicle at a signalized intersection. Moreover, the system used an interactive two-way communication between an on-board unit (OBU) and a roadside unit (RSU). The primary objective of this paper is to reduce fuel consumptions by providing the eco-speed guidance to a driver considering the driver’s acceleration/deceleration behaviour. We deployed RSU on the roadside of DGIST campus in Korea, OBU, configured a vehicle with a tablet PC, and a laptop to operate the eco-speed guidance system as shown in Figure 1. RSU controls traffic signal phases considering the eco-signal mechanism using received vehicle trajectory information and broadcasts a message periodically that contains SPaT and its location information. The laptop receives the message through OBU. Based on the received information, the laptop calculates recommended speed and displays the calculated speed and SPaT information through the tablet PC.

The main contributions of this paper are described as follows. First, we presented an eco-speed guidance system using a hybrid of eco-signal and eco-driving mechanisms. Second, we proposed a system architecture using interactive V2X communications between a vehicle and RSU and tested the system in field. Third, our proposed system reflects the driver’s acceleration/deceleration behavior on the eco-driving mechanism. Forth, we evaluated the proposed system based on fuel consumption collected from a controller area network (CAN) data using various scenarios.

The rest of this paper is organized as follows.

Section 2 illustrates the system model. The eco-speed guidance and eco-signal algorithms are proposed in Section 3. Section 4 describes the results of field tests and performance evaluation. Finally, we conclude the paper and discuss future work in Section 5.

2 ECO-GUIDANCE/SIGNAL SYSTEMS

To decrease fuel consumptions at an intersection, a vehicle is required to coordinate the speed based on the SPaT information, the current speed, and the remaining distance from the intersection. In this section, we describe an eco-speed guidance system that includes eco-driving and eco-guidance mechanisms. Note that we only consider the scenario that only one vehicle, which is equipped with OBU, is on the road. To demonstrate our proposed system in the real world, we set up a test-bed based on DSRC and conducted the field test.

2.1 System Architecture

Figure 1 illustrates the devices used in our system. RSU (LocoMate COMMANDO) is installed on the roadside. OBU (LocoMate Mini2), a tablet PC, and a laptop are mounted in a vehicle. RSU and OBU equip their own GPS antennas. We used a single signal phase (green, yellow, and red) for an approach, and the RSU controls the traffic signal.

Figure 2 shows the flow chart of proposed eco-guidance system. RSU works for two main tasks: 1) conducts a traffic signal control, 2) and extends the current remaining green time if a vehicle, which cannot cross an intersection within the green remaining time, can pass the intersection with the additional green time. In addition, RSU periodically receives vehicle trajectory information from OBU and broadcasts an a-la-carte message (ACM) that
contains SPaT information and its GPS location. As shown in Figure 3, RSU is deployed at the corner of intersection. In order to describe the current state of a traffic signal system, SPaT message is ordinarily used along with the map message in an intersection (SAE International, 2016). However, we only used ACMs for including SPaT and location information because we considered a simple scenario targeting on a single vehicle. OBU broadcasts a basic safety message (BSM) including the current GPS location and vehicle speed via V2X communication in every 0.1 second, and it transmits the current speed, the location information and the received information to the laptop via user datagram protocol (UDP).

The laptop performs for the eco-speed guidance using information from the OBU, the driver’s acceleration behavior, and a speed limit \( v_{\text{lim}} \). Then, the recommended speed and vehicle maneuver (e.g., accelerating, speed maintenance, and decelerating) are transmitted to the tablet PC, along with the current speed, SPaT, and the remaining distance information. The received information is represented on the screen like Figure 4 (a). The numbers and shading color of ‘Signal & Remaining Time’ corresponds to the current signal state and remaining signal times. When the current speed exceeds the recommended speed, the system changes the shading color to red to warn the driver as shown in Figure 4 (b).

### 2.2 Vehicle Trajectory Planning

Acceleration behavior is important because it has a critical impact on fuel consumption at a signalized intersection. In previous research (Barth et al., 2011), the best trajectory to minimize fuel consumption accelerates to a target speed quickly and then keeps the target speed until a vehicle passes an intersection completely. Besides, the best trajectory for the deceleration is to decrease the current speed to a target speed quickly, and to keep the speed until the vehicle reaches a stop bar. We used this concept to plan a vehicle trajectory in the proposed system.

In addition, we considered following cases as test scenarios. First, a vehicle maintains the current speed to pass an intersection within the remaining green time. Second, a vehicle accelerates to pass the intersection within the remaining green time. Third, a vehicle passes the intersection in next green time without a full stop when the vehicle cannot enter the intersection at the current signal. Forth, a vehicle accelerates to a target speed at yellow or red time, when the vehicle does not reach the stop-bar with the current speed until the signal changes to green, instead of a lower speed approach to the stop-bar.

### 3 ALGORITHMS

The proposed system guides eco-speeds based on SPaT information, the remaining distance from RSU, the current vehicle speed, and driver’s acceleration/deceleration behaviour. Moreover, the system also considers the eco-signal mechanism that RSU extends the remaining green time while it calculates eco-speeds. The system updates the recommended speed every second.

#### 3.1 Eco-Speed Guidance Algorithm

The notations for an Eco-speed guidance algorithm is described in Table 1. Several acceleration models have been proposed and applied in previous studies (Xia, 2013; Akcelik and Biggs, 1987; Rakha et al., 2004; Aycin and Bbnekohal, 1998) such as constant acceleration, linear-acceleration, sinusoidal acceleration, and polynomial acceleration. In this paper, we used the constant acceleration model of which the gradient is the average acceleration of driver in order to calculate a recommended speed \( v_e \). The acceleration model is presented in Figure 5 and equation (1).

\[
T(t) = \begin{cases} 
T_0 + \frac{v_e}{a} & (t \leq T_0) \\
T(t - 1) & (t > T_0)
\end{cases}
\]
In addition, there are deceleration models that are similar with acceleration models. We used the constant deceleration model of which the gradient is the average deceleration of driver in order to calculate a recommended speed when decreasing the speed to the target speed. The deceleration model is presented in Figure 5 (a) and equation (2).

$$v(t) = \begin{cases} 
-d_\beta t + v_c & (t \leq t_{tr}) \\
v(t-1) & (t > t_{tr}) 
\end{cases}$$  \hspace{1cm} (2)

The speed controls such as acceleration, speed maintenance, and deceleration depend on the expected moving distance during the specific time in every signal state. Accelerating is only allowed when a vehicle can pass the intersection within green time. For the acceleration at green, the expected maximum moving distance during $T_2$, which follows the acceleration model, needs to be longer than the distance between a vehicle and RSU ($d_{VR}$) where $T_2$ represents the remaining green time ($t_g$) or the expected remaining green time ($t_{eg}$). It is represented by $d_{m_{t2}} \geq d_{VR}$. $t_{eg}$ is the expected remaining green time to be extended according to the eco-signal algorithm. When a vehicle cannot pass the intersection within $t_g$, the vehicle anticipates that RSU will increase the remaining green time by the $\beta_{max}/2$ where $\beta_{max}$ is the maximum green extension time. The maximum moving distance at green time represents the area of graph in Figure 5 (b), which represents the vehicle accelerates from the current speed to $v_{lim}$ and then maintains the speed during the remaining green time. At yellow or red, the distance between the vehicle and the stop bar ($d_{VS}$) needs to be longer than the expected moving distance during $t_{ng}$ with the current speed ($d_{trng}$) where $t_{ng}$ is the remaining time until next green ($d_{trng} < d_{VS}$). If acceleration is allowed, $v_{r}$ is calculated using equation (3).

$$v_{r} = a_{\beta}t_{tr} + v_{c}$$  \hspace{1cm} (3)

To this end, the time to recommended speed ($t_{tr}$) as shown in equation (5) needs to be calculated first. It is calculated by equations (3) and (4). $t_{tr}$ should be shorter than $T_1$ where $T_1$ includes $t_g$, $t_{eg}$, and $t_{ng}$.

$$d = \int v(t) = (v_c + v_r) \frac{t_{tr}}{2} + (T_1 - t_{tr})v_r$$  \hspace{1cm} (4)

$$t_{tr} = \frac{T_1a_{\beta} - \sqrt{(T_1a_{\beta})^2 - 2a_{\beta}(d - T_1v_c)}}{a_{\beta}}$$  \hspace{1cm} (5)

There are two deceleration cases. First, the current speed exceeds $v_{lim}$, which is represented by $v_{lim} < v_c$. Second, a vehicle will pass a stop-bar by maintaining the current speed at red or yellow, which is represented by $d_{trng} > d_{VS}$ where $d_{VS}$ is the distance between the vehicle and a stop-bar. At the first case, $v_{lim}$ is recommended to a driver when a vehicle

<table>
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<tr>
<th>Table 1: Summary of notations.</th>
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<tbody>
<tr>
<td>$a_{\beta}$</td>
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<tr>
<td>$d_{\beta}$</td>
</tr>
<tr>
<td>$d_{VR}$</td>
</tr>
<tr>
<td>$d_{VS}$</td>
</tr>
<tr>
<td>$d$</td>
</tr>
<tr>
<td>$d_{m_{trng}}$</td>
</tr>
<tr>
<td>$d_{m_{tng}}$</td>
</tr>
<tr>
<td>$s_c$</td>
</tr>
<tr>
<td>$t_{tr}$</td>
</tr>
<tr>
<td>$t_g$</td>
</tr>
<tr>
<td>$t_{eg}$</td>
</tr>
<tr>
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</tr>
<tr>
<td>$T_1$</td>
</tr>
<tr>
<td>$T_2$</td>
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<tr>
<td>$v_r$</td>
</tr>
<tr>
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</tr>
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<tr>
<td>$dt$</td>
</tr>
<tr>
<td>$\psi$</td>
</tr>
<tr>
<td>$p^*$</td>
</tr>
<tr>
<td>$D$</td>
</tr>
<tr>
<td>$\beta$</td>
</tr>
<tr>
<td>$y$</td>
</tr>
<tr>
<td>$r$</td>
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can pass an intersection with $v_{lim}$ within the remaining green time. The other case, the recommended speed is calculated with the same way of acceleration. $t_{tr}$ in equation (8) needs to be calculated first by equations (6) and (7). $t_{tr}$ should be shorter than $t_{ng}$.

Then, $v_r$ is calculated by equation (6).

$$v_r = -d_p t_{tr} + v_c \tag{6}$$

$$d_{vS} = \int v(t) = (v_c - v_r) \frac{t_{tr}}{2} + t_{ng} v_r \tag{7}$$

$$t_{tr} = \frac{t_{ng} d_b - \sqrt{(t_{ng} d_b)^2 - 2d_b (t_{ng} v_c - d_{vS})}}{d_b} \tag{8}$$

In addition, there are two cases for speed maintenance. First, a vehicle can pass an intersection within current green time. The expected moving distance of a vehicle during $T_2$ ($d_{vS}$) with the current speed needs to be longer than $d_{vR}$. It is represented by $d_{vS} \geq d_{vR}$. Second, a vehicle cannot pass an intersection within current green time but it needs to avoid a full stop and pass a stop-bar at next green time. To this end, $d_{ng}$ with the current speed needs to be same with the distance between a vehicle and a stop-bar ($d_{vS}$). It is represented by $d_{ng} = d_{vS}$.

Note that the system cannot always avoid a full stop if $d_{vS}$ is not long enough.

Using this method, the proposed system calculates the recommended speed regarding the acceleration, the deceleration, and the speed maintenance. The details of the algorithm are described in appendix 1.

### 3.2 An Eco-signal Algorithm

RSU extends the remaining green time to decrease the fuel consumption. When a vehicle cannot pass an intersection, it can avoid a full stop, idling time, and acceleration from 0 to the target speed that cause higher fuel consumption through the extended green time.

We adopted an eco-signal mechanism proposed by Jung (2016). It aims to minimize total delays at a signalized intersection by exploiting a genetic algorithm to find an optimal signal timing plan. As described in equation (9), the optimal signal timing plan is selected among feasible signal timing plans with the number of $n$ vehicles by minimizing delay. More detailed information about the eco-signal mechanism is described in literature (Jung, 2016).

$$\min_{p \in EX, \psi \in \psi} \sum v D(p^v, \psi) \tag{9}$$

Subject to:

$$\psi = \left\{ (t_0 + \beta, y, r)_1, \ldots, (t_g + \beta, y, r)_n \right\}, 1 \leq n \tag{9.1}$$

$$p^{v-1} \leq p^v \leq p^{v+1} \tag{9.2}$$

$$0 \leq \beta \leq \beta_{max} \tag{9.3}$$

Where

- $\beta$ is green extension time;
- $\psi$ is a traffic signal timing plan;
- $p^v$ is the arrival time at the stop bar of an individual vehicle $v$;
- $D$ is the total delay (second);
- $y$ and $r$ are yellow and red intervals respectively.

Because we considered a single vehicle for field tests, we do not exploit a genetic algorithm to find the optimal value ($\beta$), which derives the minimum delay. The algorithm pseudocode is presented in appendix 2.

### 4 PERFORMANCE EVALUATION

To evaluate the proposed system with field tests, we installed RSU in DGIST campus as shown in Figure 6. The length of test-bed is 105 meters. We assumed that the test-bed consists of two intersections; a vehicle moves from the first intersection to the next intersection. The distance between a stop-bar and RSU is 15 meters. $v_{lim}$ is 40 km/h (11.12 m/s). $a_p$ is 1.7 m/s² and $a_b$ is 3.15 m/s². Green time and red time are 20 seconds and yellow time is 3 seconds. $\beta_{max}$ is 12 seconds. The vehicle used for field test is Hyundai Avante MD.

We evaluated the system performance in terms of fuel consumptions and travel-time. The exact amount of fuel injected into the engine is measured through a controller area network (CAN) in real time. The travel time to cross the intersection is calculated based on GPS information.
Table 2: Comparison of fuel consumption and travel time.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>With the guidance</th>
<th>Without the guidance</th>
<th>Fuel saving</th>
<th>Travel time saving</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fuel consumption (ml)</td>
<td>Travel time (sec)</td>
<td>Fuel consumption (ml)</td>
<td>Travel time (sec)</td>
</tr>
<tr>
<td>1</td>
<td>24.06</td>
<td>13.52</td>
<td>29.26</td>
<td>12.78</td>
</tr>
<tr>
<td>2</td>
<td>27.81</td>
<td>13.14</td>
<td>46.47</td>
<td>38.27</td>
</tr>
<tr>
<td>3</td>
<td>26.59</td>
<td>14.52</td>
<td>36.55</td>
<td>18.96</td>
</tr>
</tbody>
</table>

We compared the case with our proposed system and without case. In the without case, information of traffic lights is provided on a tablet PC without remaining signal times. We conducted three scenarios with a stopped vehicle: it starts 1) when remaining green time is 15 seconds, 2) when remaining green time is 7 seconds, and 3) when remaining red time is 12 seconds. Each scenario was conducted five times.

For each scenario, fuel consumptions and travel-time of the vehicle compared statistically using t-test. Note that the recommended speed is valid until passing the RSU. The RSU needs to send SPaT information of the downstream intersection to the vehicle for the recommended speed after the vehicle passes the RSU. Scenario 1 is to compare with/without the system when the vehicle can pass the intersection given remaining green time. Figure 7 presents vehicle movements by a time-distance graph based on GPS information. The vehicle accelerates from 0 to $v_{lim}$ or the recommended speed and then maintains the speed until passing the intersection. The case with the eco-guidance took more time (5.8%) than without case to pass the intersection as shown in Table 2 ($\alpha < 0.05$). However, fuel consumption decreased by 17.8% when the eco-guidance was applied ($\alpha < 0.05$). This is because the vehicle with the eco-guidance does not accelerate to excessive speeds as shown in Figure 10.

Scenario 2 is to test the eco-guidance when the
vehicle could not pass the intersection within remaining green time. In Figures 8 and 11, a vehicle without the eco-guidance starts to decelerate the speed from the yellow signal and waits for the green signal at the stop bar. The vehicle with the eco-guidance passed the intersection without deceleration within the extended green time. As the results, the eco-guidance significantly reduced fuel consumption by 40.16% compared with the driving without the guidance as shown in Table 2 ($\alpha < 0.05$). This is because the vehicle could pass the intersection without unnecessary deceleration. Moreover, the eco-guidance also significantly reduced travel-time by 65.7% to pass the intersection with the eco-guidance system ($\alpha < 0.05$).

Scenario 3 is the case the red time remains 12 seconds. In this scenario, the vehicle without the eco-guidance approached to the stop-bar without the information of remaining red time. As shown in Figure 9, the vehicle decelerated to the stop-bar and then accelerated when the traffic light changed to green. The vehicle with the eco-guidance accelerated from 0 to the recommended speed and maintained the speed during the remaining red time. As shown in Table 2, the eco-guidance reduced fuel consumption and travel time when compared without case by 27.3% and 23.4% ($\alpha < 0.05$), respectively. This is because the vehicle does not need to decelerate while approaching to the intersection during it followed eco-guidance information.

Because the participants complied the eco-guidance very well, vehicle speeds with eco-guidance and recommended speed by eco-guidance were similar as shown in Figures 7, 8, and 9.

5 CONCLUSION AND FUTURE WORK

In this paper, we proposed an eco-speed guidance system using a hybrid of eco-driving and eco-signal mechanisms. Our system guides the recommended speed to a driver based on driver acceleration/deceleration behavior, SPaT information, and the remaining distance from the intersection. We evaluated our proposed system with field tests using communication devices (e.g., DSRC) in terms of fuel consumption collected via CAN data and travel time. As a result, we found that the proposed system contributes to reduce fuel consumption and travel time when a driver complied eco-guidance information.

In the near future, we will further investigate the effect of multiple vehicles on the eco-guidance and the safety critical issues and improve our system to cover the more complicated situation on the vehicles, which partially follow the guidance, in field. Moreover, we will consider multiple intersections in a wide test region to test various scenarios and the more accurate vehicle localization to calculate the precise recommended speed to overcome GPS errors.

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### APPENDIX I

**Pseudocode of eco-speed guidance algorithm**

if green signal then
  if $d_{vb}/t_g \leq v_c$ and $v_c \leq v_{lim}$ then
    $s_c \leftarrow$ speed maintenance;
    $v_r \leftarrow v_c$;
  end if
else if $d_{vb}/t_g \leq v_c$ and $v_c > v_{lim}$ then
  $s_c \leftarrow$ deceleration;
  $v_r \leftarrow v_{lim}$;
else if $d_{nst} \geq d_{vb}$ then
  $s_c \leftarrow$ acceleration;
  $v_r \leftarrow a_b \cdot t_r + v_c$;
else if $d_{nst} \geq d_{vb}$ then
  if $v_c < v_{lim}$ then
    $s_c \leftarrow$ acceleration;
    $v_r \leftarrow a_b \cdot t_r + v_c$;
    if $v_r = v_c$ then
      $s_c \leftarrow$ speed maintenance;
      $v_r \leftarrow v_c$;
    end if
  end if
else if $v_c = v_{lim}$ then
  $s_c \leftarrow$ deceleration;
  $v_r \leftarrow v_{lim}$;
end else if
else if $v_c = v_{lim}$ then
  $s_c \leftarrow$ speed maintenance;
  $v_r \leftarrow v_c$;
else 
  $s_c \leftarrow$ deceleration;
  $v_r \leftarrow v_{lim}$;
end else if
else 
  $v_r \leftarrow v_{lim}$;
  if $v_c < v_r$ then
    $s_c \leftarrow$ acceleration;
  end if
else if $v_c = v_r$ then
  $s_c \leftarrow$ speed maintenance;
  $v_r \leftarrow v_c$;
else if $v_c < v_r$ then
  $s_c \leftarrow$ acceleration;
  $v_r \leftarrow v_{lim}$;
end else if
else 
  $v_r \leftarrow v_{lim}$;
  if $v_c < v_r$ then
    $s_c \leftarrow$ acceleration;
  end if
end if
end else if
end if
end for

### APPENDIX II

**Pseudocode of eco-signal algorithm**

for $0 \leq i \leq \beta_{max}$ then
  $t_g \leftarrow t_g + i$;
for $n$ vehicles then
  if $d_{nst} \geq d_{vb}$ then
    $dt[i] \leftarrow dt[i]$;
  end if
  else
    $dt[i] \leftarrow dt[i] + t_g + y + r$;
  end else
end for
if $dt[i-1] > dt[i]$ and $i > 0$ then
  $\beta \leftarrow i$;
end if
end for