Highway Reservation Strategy: Analytical Modeling Approach

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Abstract: Inspired by the success of reservation systems in airlines industries, and the Connected Vehicle technology supporting vehicular communications, this paper investigated a highway reservation and developed a mathematical optimization formulation to solve the optimal trip scheduling plan for a traffic network. The performance was quantified by total monetary cost of travel time and applicable early arrival time or late arrival time. In the two numerical case studies with an assumption of 100% compliance of the users to the reservation system’s scheduling, the system cost was 24.1% and 21.7% lower than those of the two corresponding user equilibrium solutions. The reservation system effectively redistributed the peak hour demand to the non-peak hours by limiting the reservation maximum flow rate of the reservation links.

1 INTRODUCTION

Metropolitan transportation road networks are typically congested due to concentrated travel activities and consequently faced with increased travel times, air pollution, noise, and traffic crashes. As shown by the annual person-hours of highway traffic delay per auto commuter, between 1982 and 2014, provided by the 2015 National Transportation Statistics (Bureau of Transportation Statistics, 2015), congestion has increased substantially over the 30 years. The delay per commuter in 2014 was 42 hours. In the very large urban areas (3 million and over population), the average auto commuter delay is 63 hours. Adding more capacity by providing more road lanes and more public transportation is the most fundamental congestion solution in most growing urban regions to satisfy the increasing travel demand. However, transportation system capacity almost always increases at a slower rate than the demand growth. As shown by the Road Growth and Mobility Level Exhibit (Schrank et al., 2012), 56 in 101 study areas have travel demand growth 30% faster than supply, and only 17 areas have a less than 10% gap between demand and supply growth. Even if the capacity growth perfectly matches with travel demand, new problems would occur as reduction in congestion induces departure time shifts into peak-hour (Hendrickson and Plank, 1984). In addition, crashes or work zones may create bottlenecks on the highway and seriously downgrade the highway capacity. While the Intelligent Transportation Systems helped mitigating the congestion impact by providing solutions to efficient use of highway systems, transportation system can benefit from a new innovative approach to address congestion problem.

Chow found that the necessary condition for transportation system optimum is having the inflow equal to the bottleneck capacity for all routes and all departure time intervals in use (Chow, 2009). This requires dispersing the peak hour travel demand by time, and can be realized by adopting a highway reservation concept (Edara and Teodorovic, 2008; Su et al., 2013). Travelers in such a reservation system need to book in advance for the right of using the highway segments during their desired time. If some time slots have been fully booked, additional travelers need to book an alternative time or route. A major difference between reserving airline seats and highway slots is that an airline seat is a well-defined object that is clearly identifiable, but a highway slot is difficult to define in practice. The travelers need to be shown the “edges” of a slot in time and space, and to be indicated of admittance into the system as well as being notified of violations. While existing transportation system is not likely to handle highway
reservation system due to lack of real-time communications and computation power, the connected vehicle technology would make the highway reservation idea feasible.

A proof-of-concept simulation study was conducted to investigate the potential benefits of the highway reservation system (Su et al., 2013). If a time interval is fully booked, the booking center will recommend other intervals in close proximity, and the travelers choose which one to accept. This algorithm was applied to a carefully designed microscopic simulation testbed and the reservation scenario outperformed the baseline in terms of total delay time and emissions.

In this paper, we developed a highway reservation system using an analytical modeling approach, and solved the optimal trip plans using an analytical scheduling algorithm. The remainder of the paper is organized as follows: “2 Literature Review” section briefly discusses previous studies and concepts on highway reservation and departure time choice problem. “3 Model” section describes a big picture, the system objective, solution approach and two case studies of the proposed analysis approach, followed by “4 Conclusions and Future Research” section.

2 LITERATURE REVIEW

The concept of road reservation or trip-booking is mentioned in the literature as early as 1990s, but extensive modeling efforts have not been done till the recent 5 years. Some researchers conducted surveys to explore travelers’ acceptance of the reservation system and its effectiveness (Akahane and Kuwahara, 1996; Kim and Kang, 2011). Akahane and Kuwahara found, if the participation compliance rate is 90%, a 15-minute adjustment of the departure time could eliminate congestion over single bottleneck (Akahane and Kuwahara, 1996). Kim and Kang found that 73.4% of the respondents would participate in or accept if an expressway reservation system is implemented during South Korea’s national holiday (Kim and Kang, 2011). Wong (Wong, 1997), Iftode and Gerla et al. (Gerla and Iftode, Undated; Iftode, Smaldone, Gerla, and Misener, Undated; Ravi, Smaldone, Iftode, and Gerla, Undated) pioneered the discussions of the basic functions, advantages and difficulties of a highway booking system. Wong suggested slicing the highway capacity into time intervals on which trip bookings are based. Several researchers (Gerla and Iftode, Undated; Iftode et al., Undated; Ravi et al., Undated) proposed the coexistence of reserved lanes with general-purpose lanes, so that opted-out or rejected users can always use the general-purpose lanes. A merging/diverging assistance system is needed because of the lane separations.

Koolstra was the first that brought the scheduling cost into the highway reservation system (Koolstra, 2000). They evaluated the queuing and scheduling costs with single bottleneck and heterogeneous travelers. They found all queuing costs can be eliminated without increasing the average rescheduling costs. Their study also supported that a freeway reservation might be more effective in practice than road pricing. And reservation system with variable booking fees is an option to incorporate the benefits of congestion pricing. Veeraraghavan et al. showed, from the standpoint of queuing theory, that a reservation system is necessary to avoid waiting, when the average waiting time is large at the optimal point of operation (Veeraraghavan et al., 2009). De Feijter et al. stated that the objective of trip-booking is improving reliability and predictability of travel times, and his simulation experiments showed exactly so (de Feijter et al., 2004). Edara and Teodorovic took the lead in conducting extensive modeling work of reservation system by proposing a Highway Allocation System (HAS) and Highway Reservation System (HRS) (Edara and Teodorovic, 2008). HAS selects trips from received booking requests to maximize the total Passenger-Miles-Traveled over a period. HRS works in an on-line mode to decide whether a request should be accepted or rejected. Edara et al. showed that HAS produced 35% to 45% more Passenger-Miles than the two ramp-metering algorithms (Edara et al., 2011).

Different from the idea of sliced highway capacity by time, Wong and Liu et al. proposed a token-based reservation idea (Wong, 1997; Liu et al., 2013). Each road segment has a set of tokens, and the number of tokens is the product of the segment length and critical density, or the total number of vehicles on the link when the capacity condition occurs. A reservation request is accepted only if at least one token on the requested segment is available, and the requested time slot does not overlap with any of the existing reserved time slots on this token. Greenshield’s linear speed-density model is used in Liu’s study, thus the optimal density is a half of jam density, and optimal speed is a half of free flow speed. Liu’s work is a meaningful exploration of different ways of modeling the reservation system.

The highway reservation system works by rescheduling the travelers’ departure time as well as route choice to avoid over-capacity traffic flows. Thus, the departure time choice modeling methods are useful for this study. The most commonly used travel time model is Vickrey’s bottleneck link model (Vickrey, 1969). This has been used in numerous departure time studies (Arnott et al., 1990; Chow,
2009; Hendrickson and Kocur, 1984; Huang and Lam, 2002). Hendrickson and Kocur analyzed the users’ departure time decisions in a single bottleneck under three different settings (Hendrickson and Kocur, 1984). Arnott et al. studied user equilibrium, system optimum and various toll regimes for a network with parallel routes between one OD pair (Arnott et al., 1990); Huang and Lam solved a user equilibrium with route and departure time choices (Huang and Lam, 2002). Other than Vickrey’s model, Mahmassani and Herman (Mahmassani and Herman, 1984) used Greenshield’s traffic flow relationship in an ideal arterial to represent congestion effects. This model works only for routes with single uninterrupted link, as it is difficult to calculate the exact exit flow rate.

Some other studies developed discrete choice models based on survey data to see what factors can affect travelers’ departure time choices (Hendrickson and Plank, 1984; Robert and Small, 1995; Small, 1982). Small’s work (Small, 1982) is the very first econometric study of the trip scheduling behaviors at the individual level. The discrete logit model of the commuters’ work trip scheduling provides useful information of time values, the relative magnitude of them is consistent with Hendrickson and Plank (Hendrickson and Plank, 1984): late arrivals at work have the highest value of time, early arrivals have the lowest, and the value of wait time on the road is between them. Noland and Small (Robert and Small, 1995) analyzed the effect of uncertain travel times on the commuting departure time choice. They found that travel time uncertainty can account for a large proportion of the morning commute cost. A few researchers analyzed theoretically the dynamic traffic assignment problem with departure time choice (Wie et al., 1995; Friesz and Mookherjee, 2006; Chow, 2009). Wie et al. (1995) formulated the user equilibrium and system optimum conditions and compared the two using a numerical example.

Under system optimum, travelers with different departure time might have different total cost, and they have incentive to adjust departure time and arrive at user equilibrium. Some researchers (Hendrickson and Kocur, 1984; Hendrickson and Plank, 1984; Vickrey, 1969) suggest using time dependent tolls to help balance the unequal total cost, so that different departure time will generate the same cost. With the optimization model’s results provided in this paper, the exact time dependent toll pattern can also be identified. This toll idea works under two conditions: the exact travel demand pattern is known, and all the travelers are homogeneous. However, neither of the two is satisfied in practice.

3 MODEL

3.1 Big Picture

According to two economic studies of commuters’ traveling behavior (Hendrickson and Plank, 1984; Small, 1982), late arrivals at work have the highest value of time, early arrivals have the lowest, and the value of wait time on the road is between them. That means, if there is anticipated congestion, the commuters have the incentive to depart earlier (also arrive earlier) to avoid the congestion. Highway reservation system provides a reliable mechanism for them to do so. Another advantage of highway reservation system is reducing the travel time uncertainty, as “travel time uncertainty can account for a large proportion of the morning commute cost” (Robert and Small, 1995). These economic studies lay the foundation for the highway reservation system.

To provide a proper “edge” of the reservation token to the user, highway system is divided into multiple links by on- and off-ramps, and time is discretized into intervals with link capacity sliced (Wong, 1997). A reservation slot is defined as the combination of several consecutive links and time intervals. For example, a user can reserve a 3 mile-long segment (may have multiple links) between time 8:30 am and 8:33 am. Certain tolerance could be defined by the local traffic conditions to accommodate inaccurate travel time estimate. For example, ±5 minutes tolerance could be used if the local traffic is unpredictable. This segment’s operational speed is set to be 60 mph. That’s why travel time is 3 minutes. Such accurate arrival time and speed control would be feasible by transmitting speed and lane-change advisories messages from the operation center with Connected Vehicle technology.

The proposed highway reservation system works by redistributing the peak hour travel demand earlier or later to non-peak hours. Its validity depends on how the users respond. Some of them may have flexible schedule and are willing to accept any rescheduling, while some of them may not cooperate. The users’ attitude depends on a lot of factors, such as work schedule flexibility, experience with the reservation system, etc. In this paper’s model, it is assumed that the highway users will fully cooperate with the booking center, meaning they accept any rescheduling, and will travel by the planned schedule. Another assumption is that all the lanes on the highway are reserved in this paper’s model.

Compared with HOV lane usage strategy, reservation system produces higher utilization of the highway capacity when the demand level is low, as there might not be enough vehicles to occupy the HOV lane. HOT strategy might be able to lower the
tolls to make better use of the capacity, but the elasticity of the demand to the toll is difficult to be estimated. Sometimes it could be too late to increase the tolls to avoid congestion if the travel demand bumps up. All these challenges do not exist in the reservation system. In a sense, it makes the traffic information transparent to both demand and supply side beforehand.

3.2 System Objective

All the notations used in the models are listed in Table 1 of Appendix. The objective of the reservation system is minimizing the total cost of its users, a weighted sum of early arrival or late arrival cost and travel time cost (Equation 1). While this appears to be similar to departure time choice model, the main difference is that the proposed research system ensures reliable travel time along the roadways within the reservation system by enforcing the capacity constraints. The decision variable is \( V_{ijklr} \). The \( C_{ijklr} \) is calculated by the Successive-Update approach mentioned in the next section of the paper, and there is explicit expression for it. So the objective function in Equation 1 is just for illustration purpose. In implementation, we could remove indices of \( i, j \) and \( r \), when all possible routes are identified and indexed.

\[
\text{Min} \sum_{i=1}^{O} \sum_{j=1}^{D} \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{r=1}^{R} V_{ijklr} C_{ijklr} \quad (1)
\]

\( C_{ijklr} = w_1 \times \max(DAT_i - AAT_{ijklr}, 0) + w_2 \times \max(AAT_{ijklr} - DAT_i, 0) + w_3 \times \text{TravelTime}_{ijklr} \)

Subject to:

\[
\sum_{k=1}^{K} \sum_{l=1}^{L} V_{ijkl} = \text{Demand}_{ijl} \quad \text{for all } i, j \text{ and } l
\]

\( \text{Inflow}_{ikl} < \text{Capacity}_{ikl} \quad \text{for all } k \text{ and } l \)

\( V_{ijklr} > 0 \) and integer \( \text{for all } i, j, k, r \text{ and } l \)

Where,

\[ V_{ijklr} \text{ = The number of trips between OD } (i, j) \text{ with desired arrival time DAT}_i \text{ using route } r \text{ that start the trip from the } k_0 \text{ time interval} \]

\[ C_{ijklr} = \text{Total travel cost of a trip between OD } (i, j) \text{ with desired arrival time DAT}_i \text{ using route } r \text{ that start the trip from the } k_0 \text{ time interval} \]

\[ \text{Inflow}_{ikl} = \text{The inflow of link } l_k \text{ in the } k_0 \text{ time interval} \]

\[ \text{Capacity}_{ikl} = \text{The capacity of link } l_k \text{ in the } k_0 \text{ time interval} \text{ (currently it does not change by time)} \]

\[ \text{DAT} = \text{Desired Arrival Time} \]

\[ \text{AAT} = \text{Actual Arrival Time} \]

We adopted Vickrey’s Model (Vickrey, 1969) for the link behaviors. It is a deterministic queuing model that considers each link to be free flowing with a constant travel time, and a bottleneck at the beginning or end of the link with fixed capacity. Delays will occur when the traffic inflow continuously exceeds the capacity for a substantial period. If there is no queue, the outflow rate is equal to the inflow rate, and the travelers have no delay. It assumes relatively stable inflows, without considering stochastic variations. Vickrey’s queue model is selected in this study because 1) the maximum flow rate can be considered explicitly and 2) it is easy to calculate the exit flow rate and rate, and propagate the exit flow into the successor links. The queue length evolves as shown in (2) (Huang and Lam, 2002). When \( \lambda_a(k) \) is higher than \( \mu_a \), the capacity of link \( a \), the queue length increases from \( q_a(k-1) \) to \( q_a(k) \), and if \( \lambda_a(k) \) is lower than \( \mu_a \), the queue length decreases.

\[ \Delta t \times \lambda_a(k) \text{ is the number of vehicle arrived at link } a \text{ in time interval } k. \text{ The exit time of these vehicles from link } a \text{ and the associated exit flow rate depend on the current queue length and the relative magnitude of } \lambda_a(k) \text{ and } \mu_a. \]

\[ q_a(k) = \max[q_a(k-1) + \Delta t(\lambda_a(k) - \mu_a), 0] \quad (2) \]

\( \lambda_a(k) \text{: Inflow rate of the } k^{th} \text{ time interval} \]

\( \mu_a \text{: Capacity of the link segment } a \)

\( t_0 \text{: Travel time under “typical” speed} \]

3.3 Successive-Update Approach

The link bottleneck model can calculate exit time and rate from each link. The exited vehicles enter the successor link, together with vehicles from other routes that also use the successor link. The successive-update approach uses an INFLOW vector and OUTFLOW vector to keep the flow information
for each link, and updates them in each time step, until all the vehicles have reached their destination. The capacity constraint is realized by including a penalty term in the objective function when the inflow rates exceed capacity. The routes between each OD pair are predetermined either manually (e.g., identifying commuters’ habitual routes by analyzing their routes over adequate time period) or by a route-searching algorithm (e.g., k-shortest path algorithm), and stored in ROUTES, an R by m matrix, where R is the total number of routes. m is the maximum number of links in a route. All the routes are numbered by the row ID in ROUTES, no matter which OD pair they connect. The initial traffic assignment is stored in INPUT, an R by K by DAT matrix. Note that it is assumed that the users’ desired arrival time is not continuous but belongs to a set of discrete time points, as they are determined by morning commuters’ work start time, which is not continuous most of the time.

INFLOW_r and OUTFLOW_r record the flow propagation process for link a. They are 2K by R matrices. 2K is used because the propagation process runs for 2K time intervals, in case some trips cannot finish at the end of Kth interval. For the links that are the beginning of any route, their INFLOW matrices are initialized using INPUT. For example, if link b is the first link of route r, INFLOW_r(r, k) is initialized by summing up INPUT(r, k, 1:DAT). During the traffic propagation process, in each time step k, sum(INFLOW_r(1:K, k)) vehicles enter link a, and OUTFLOW_r is updated according to the calculated exit flow time and rate based on Vickrey’s model. To maintain flow conservation, at the end of each time step, INFLOW_r(r, 1:2K) of all the links are updated by taking in vehicles from the predecessor links’ OUTFLOW. A QUEUE_r vector records the queue length of link a in all the time intervals. A DEPART_r vector records the flow exit time of link a. The time interval is set to be shorter than the shortest travel time of all the links, so that the outflow of the links will never affect the successor’s inflow in the same time interval. When the propagation process is finished, the DEPART vectors have the exit time of the trips from each link. By tracking down the DEPART vectors of the links on route r, we obtain the arrival time at the final destination of the vehicles using route r. With the final arrival time, the system objective is calculated.

3.4 Solving the Optimization Problem

This study adopted the Interior Point Method (IPM) (Nocedal and Wright, 2006). Since there is no close-form formula, the algorithm used finite-difference equation to find the search direction. Given an initial solution, the algorithm began the iterative process to search for the next solution. The initial solution assumed that the demand is evenly distributed in all the routes and all the time intervals.

3.5 Numerical Example

This paper uses a numerical example illustrated in Huang’s study (Huang and Lam, 2002). Huang solved the user equilibrium route and departure time choice problem. Thus, using the same example makes it consistent to compare the performance of the highway reservation system with user equilibrium solution.

The grid network, as shown in Figure 1, includes nine nodes, 12 links and two OD pairs (from A to C and from B to C). All the typical travel time and capacity of the links are shown in the figure. The trip demands from A to C and from B to C are 20,000 and 10,000 veh, respectively. All the other settings are the same with the previous example. The network is symmetric as well as the input data, so there are only three unique routes: 1 (6 is the same with 1), 2 (3, 4 and 5 are the same with 2), and 7 (8 is the same with 7). The program treated all the routes independently, and symmetric outputs are indeed found.

Using IPM, the optimality condition was satisfied after one hour run. The computation time is a topic of future research when a real size network and travel demand is dealt with. Figure 2 shows the inflow rate of the three unique routes. There are no trips on route 1 (6), because route 1 (6) have longer travel time than route 2 (3, 4, 5), and all of them share bottleneck links 6 and 12. This model has the potential of identifying critical links and under-utilized links. Figure 3 shows the traffic flow rates of six unique links. It is noted that links 2 (10) and 5 (9) have no traffic at all, and link 6 (12) has reached capacity. This is easy to understand since all trips ending in zone C need to use either link 6 or link 12. All other links have some traffic but not saturated. This is an evidence that links 6 and 12 are the bottlenecks in this grid network.

A total of 23,909 vehicles arrive earlier than 9 am, and the average early arrival time is 0.855 hr. A total of 6,091 vehicles arrive later than 9 am, and the
average late arrival time is 0.218 hr. The average cost of all the vehicles between A and C is about 7.91 dollars, and 8.03 dollars between B and C. Huang’s user equilibrium average cost is about 11 dollars between A and C, and 7 dollars between B and C (Huang and Lam, 2002). Although B-to-C distance is shorter than A-to-C, the B-to-C traveler average cost is higher than A-to-C travelers. This is clearly shown in Figure 2 that some of the trips on route 7 (B-to-C) arrive late, and the late arrival cost is much higher than early arrival and travel time.

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Figure 2. Optimized Traffic Flow of Three Unique Routes [note: Route 1 has no traffic].

Figure 3. Optimized Traffic Flow of Six Unique Links [note: Links 2 and 5 have no traffic]

3.6 Discussions

The proposed model sliced the highway capacity into time intervals as proposed by Wong (Wong, 1997). It is noted that the token-based approach (Liu et al., 2013) is not adopted because 1) it is difficult to determine the time each token is occupied, and 2) too many overlapped tokens being reserved will lead to short-term excessive demand not properly handled. Even though coexistence of reserved lanes and general-purpose lanes is not modeled explicitly in the paper, the model can be modified to reflect this coexistence, e.g. setting the reservation flow rate as capacity of a single lane. In the co-existing lanes scenario, controlling the speed differential is critical to enable lane-changes occurring in a safe manner, otherwise it would be difficult to enter and exit the reserved lanes. While a previous study of Koolstra (2000) that evaluated a single bottleneck, our model is capable of simulating multiple connected links being reserved and is scalable. Unlike Edara and Teodorovic (2008), our model explicitly considers the scheduling cost that incorporates the impacts of departure time changes.

Given 100% compliance rate is used, the results should be treated as an “up-ceiling” of the reservation system’s benefits. To consider realistic compliance rate, one could implement auction based reservation system (Su and Park, 2015). To ensure efficient speed operations in the reserved lane, one could consider a cooperative adaptive cruise control (CACC) technology (Park et al., 2011; Schakel et al., 2010).

4 CONCLUSIONS AND FUTURE RESEARCH

This paper proposed an innovative highway reservation system as a travel management strategy, and formulated and solved it as an optimization model. This model is capable of finding an optimal scheduling plan that the reservation system could make for optimal system performance, under a constraint that all the links are operated below the capacity level. In two case studies, by applying the reservation concept over highway networks, the total monetary costs reduced by 20% to 25%, comparing with user-equilibrium traffic assignments. Given the optimization model works under the assumption that all the users are fully compliant with the scheduling plan, it is recommended the future research should consider an agent-based modeling approach to consider diverse user behaviors.

A few critical issues related to implementation are discussed. Another main challenge is how to handle non-recurrent congestions due to crashes or incidents. A few strategies that would help mitigate include (i) activating reserved capacity (that is saved for emergency vehicles), (ii) accepting no more on-the-fly reservations, (iii) providing incentives to drivers willing to give up their near future reservations, and (iv) implementing route guidance system to divert the demand.
ACKNOWLEDGEMENTS

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REFERENCES


APPENDIX

Table 1: Notations used in the Link Bottleneck Model and Successive-Update Approach

<table>
<thead>
<tr>
<th>Notation</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>Integer</td>
<td>Index of origin, a member of {1, 2, ... O}</td>
</tr>
<tr>
<td>j</td>
<td>Integer</td>
<td>Index of destination, a member of {1, 2, ... D}</td>
</tr>
<tr>
<td>k</td>
<td>Integer</td>
<td>Index of time interval, a member of {1, 2, ... K}</td>
</tr>
<tr>
<td>r</td>
<td>Integer</td>
<td>Index of a route in (R_{ij}), a member of {1, 2, ... (R_{ij})}</td>
</tr>
<tr>
<td>l</td>
<td>Integer</td>
<td>Index of a desired arrival time, a member of {1, 2, ... DAT}</td>
</tr>
<tr>
<td>a</td>
<td>Integer</td>
<td>Index of a link, a member of {1, 2, ... A}</td>
</tr>
<tr>
<td>m</td>
<td>Integer</td>
<td>The maximum number of links of all the routes</td>
</tr>
<tr>
<td>n</td>
<td>Integer</td>
<td>Each link has a number of routes that start from it. n is the largest number.</td>
</tr>
<tr>
<td>O</td>
<td>Integer</td>
<td>Total number of origins</td>
</tr>
<tr>
<td>D</td>
<td>Integer</td>
<td>Total number of destinations</td>
</tr>
<tr>
<td>K</td>
<td>Integer</td>
<td>Total number of time intervals</td>
</tr>
<tr>
<td>(R_{ij})</td>
<td>Integer</td>
<td>Total number of routes between OD ((i, j)). (R_{ij}) is a subset of (R).</td>
</tr>
<tr>
<td>R</td>
<td>Integer</td>
<td>Total number of routes between all the ODs pairs</td>
</tr>
<tr>
<td>DAT</td>
<td>Integer</td>
<td>Total number of desired arrival times</td>
</tr>
<tr>
<td>A</td>
<td>Integer</td>
<td>Total number of links in the network</td>
</tr>
<tr>
<td>(w_1)</td>
<td>Double</td>
<td>Value of time for the early arrival</td>
</tr>
<tr>
<td>(w_2)</td>
<td>Double</td>
<td>Value of time for the late arrival</td>
</tr>
<tr>
<td>(w_3)</td>
<td>Double</td>
<td>Value of time for travel time</td>
</tr>
<tr>
<td>(V_{ijkt})</td>
<td>Integer</td>
<td>Number of vehicles between OD ((i, j)) with desired arrival time (DAT_{ij}) that travel on route (R_{ij}). (R_{ij}) is one of the routes in (R) and start in the (k)th interval.</td>
</tr>
<tr>
<td>(C_{ijkt})</td>
<td>Double</td>
<td>Average cost of the vehicles (V_{ijkt})</td>
</tr>
<tr>
<td>(AAT_{ijkt})</td>
<td>Double</td>
<td>Actual arrival time of the vehicles (V_{ijkt})</td>
</tr>
<tr>
<td>ROUTES</td>
<td>R by m matrix</td>
<td>Each row represents a route's links.</td>
</tr>
<tr>
<td>GP</td>
<td>General Purpose Lane</td>
<td>-</td>
</tr>
</tbody>
</table>