Eco approaching at an isolated signalized intersection under partially connected and automated vehicles environment

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A R T I C L E   I N F O

Article history:
Received 18 August 2016
Received in revised form 23 February 2017
Accepted 2 April 2017

Keywords:
Eco-driving
Isolated signalized intersection
Partially connected and automated vehicles environment
Mobility
Fuel efficiency
Speed optimization

A B S T R A C T

This research proposed an eco-driving system for an isolated signalized intersection under partially Connected and Automated Vehicles (CAV) environment. This system prioritizes mobility before improving fuel efficiency and optimizes the entire traffic flow by optimizing speed profiles of the connected and automated vehicles. The optimal control problem was solved using Pontryagin’s Minimum Principle. Simulation-based before and after evaluation of the proposed design was conducted. Fuel consumption benefits range from 2.02% to 58.01%. The CO2 emissions benefits range from 1.97% to 33.26%. Throughput benefits are up to 10.80%. The variations are caused by the market penetration rate of connected and automated vehicles and v/c ratio. No adverse effect is observed. Detailed investigation reveals that benefits are significant as long as there is CAV and they grow with CAV’s market penetration rate (MPR) until they level off at about 40% MPR. This indicates that the proposed eco-driving system can be implemented with a low market penetration rate of connected and automated vehicles and could be implemented in a near future. The investigation also reveals that the proposed eco-driving system is able to smooth out the shock wave caused by signal controls and is robust over the impedance from conventional vehicles and randomness of traffic. The proposed system is fast in computation and has great potential for real-time implementation.

1. Introduction

As reported by U.S. Environmental Protection Agency, the transportation sector is the second largest source of greenhouse gas (GHG) emissions and has contributed about 26% of total U.S. GHG emissions in 2014 (Sources of Greenhouse Gas Emission, 2016). Improving fuel efficiency and reducing emissions have become a critical focus of transportation research. Many approaches have been proposed accordingly, including advanced vehicle technology (Mendez and Thirouard, 2008), Eco Fuel (Durbin et al., 2011), traffic demand management (Strompen et al., 2012), advanced traffic signal control (Chen et al., 2011) and vehicle operation (Barth et al., 2011; Hu et al., 2016). Eco-driving is one important component of fuel efficiency improvement technologies (Barth et al., 2011; Xia et al., 2012). The main idea is to reduce acceleration, deceleration and idling by optimizing vehicle speed profile (Xia et al., 2013a,b; Li et al., 2014). Eco-driving can be categorized into:

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http://dx.doi.org/10.1016/j.trc.2017.04.001
0968-090X/Published by Elsevier Ltd.
ecoco-driving on freeway (Hu et al., 2016; Saboohi and Farzaneh, 2009; Barth and Boriboonsomsin, 2009; Wang et al., 2014) and eco-driving on signalized arterial (Barth et al., 2011; Xia et al., 2013a; Xia et al., 2013b). The scope of this research is on eco-driving on signalized arterials.

A number of eco-driving strategies on signalized arterials have been proposed. Some developed optimal controllers for individual vehicles, providing optimal speed profiles. Mandava and Barth designed a speed advisory system for human drivers traveling on signalized arterials under light traffic condition (Mandava et al., 2009). Later, Barth upgraded his speed advisory system with real-time capability (Barth et al., 2011; Xia et al., 2013a,b). Liu developed a more advanced controller that could consider the effect of front queue (He et al., 2015). Some others worked on providing ecological speed profiles for accelerating to desired speed when leaving an intersection (Xia et al., 2013a; Xia, 2014; Hao et al., 2015). Others studied to reduce the total fuel consumption and emissions by improving throughput (Zhou et al., 2017; Ma et al., in press; Lee and Park, 2012). Lee and Park removed signal with the help of Connected and Automated Vehicle (CAV) technology to improve throughput and reduce stops and deceleration delay which significantly brought down fuel consumption and emissions (Lee and Park, 2012). Li designed a control strategy based on shooting heuristic (SH) algorithm to pre-cluster vehicles into tight and fast marching platoons before passing an intersection in order to maximize throughput (Zhou et al., 2017; Ma et al., in press).

Non-connected vehicles can be “controlled” indirectly when they are following a connected vehicle. In this case, car-following model could be applied to establish “cooperation” between CAV and non-connected vehicles. Car-following model is a well-accepted concept that has been studied for years (Mehmood et al., 2003) and has been verified on arterials with field data (Ahn et al., 2004). For instance, it has been utilized to infer driver’s intent at signalized intersection (Liebner et al., 2012), or used to analyze traffic oscillation under congestion (Li et al., 2014; Li and Ouyang). The concept of applying car-following model to indirectly “control” non-connected vehicles has also been confirmed valid in past studies. Kamal utilized the Gipps’ car-following model to predict the state of the conventional vehicle to support the optimization of autonomous vehicle (Kamal et al., 2015). Wang used the Helly car-following model to construct a cooperative cruising system on freeway (Wang et al., 2014). Therefore, even in a partially CAV environment, a control system can be established.

The existing eco-driving systems have limitations. First, they are not suitable for congested signalized intersection (Barth et al., 2011; Xia et al., 2013a,b; Kamal et al., 2015; Rakha and Kamalanathsharma, 2011), because these eco-driving systems tend to slow vehicles down and thus have a negative impact on the throughput. As the result, although the fuel efficiency of the few CAVs are improved, the vast majority of the traffic is sacrificed. Second, they are not fit for real implementation in the current world. Most of them assumed that all vehicles are connected and automated vehicles which can take on the entire dynamic driving task (Zhou et al., 2017; Ma et al., in press; Lee and Park, 2012). Unfortunately, the market penetration of connected and automated vehicles will not reach one hundred percent until 2060s (Alessandrini et al., 2015). Hence, the technologies developed based on complete connected and automated vehicles environment are not practical in a very long time.

Therefore, the objective of this research is to develop an optimal controller that is:

- functional in a partially connected and automated vehicles environment (feasible for real-world implementation in the near future)
- prioritizing mobility before improving fuel efficiency
- fast enough for potential real-time implementation
- applicable for isolated signalized intersection

The reminder of the paper is organized as follows: Section 2 ‘Control Structure’ provides high-level descriptions and highlights of the control structure; Section 3 ‘Mathematical Formulation’ presents problem formulation and the associated solution; Section 4 ‘Simulation Evaluation’ describes simulation set-up and the associated results; Section 5 ‘Conclusion’ entails the conclusions and future works.

2. Control structure

The goal of the proposed optimal controller is to improve fuel efficiency for vehicles approaching an isolated signalized intersection while causing no adverse effect to throughput. There are two highlights of this proposed controller:

- Eco-driving with mobility priority: The proposed controller provides the most ecological driving speed advisory while maintaining optimal mobility status for the intersection. The controller puts mobility (throughput) a higher priority than ecology. It enforces the CAV’s final condition to optimize the mobility. The design ensures that all CAVs pass through the intersection with the smallest headway with their preceding vehicle and travel at legal speed limit. From a congregated perspective, the controller forces vehicles into tightest and fastest marching platoons at the stop line before optimizing vehicles fuel efficiency. Since flow rate equals to the product of density and speed, by maximizing density and speed, throughput at an intersection is optimized. Therefore, users of this proposed controller do not sacrifice travel time to save gasoline. It is a pure “win” control system.
“Control” non-CAVs: One important idea of the proposed controller is to optimize the entire traffic flow (both CAVs and conventional non-connected vehicles) by optimizing speed profiles of the CAVs. The controller is designed for vehicles approaching an isolated intersection under mixed traffic condition. The mixed traffic consists of two types of vehicles. One is informed vehicles (CAV) which could communicate information with signal control, transmit vehicle state to and receive speed advisory from the optimal controller. The other type is uninformed vehicles which are conventional vehicles in the current real world. It should be noted that uninformed vehicles have their travel pattern (car-following model). They can be a part of a controlled system, even though they cannot be controlled directly.

The eco-driving system requires the following equipment: (i) one loop detector installed upstream of the interested intersection which detects the arrival of all vehicles; (ii) communication device installed both on CAVs and traffic lights to enable communication between vehicles and infrastructures. The following assumptions are made:

- Lane-changing and overtaking maneuvers are not allowed.
- No communication issues—such as delay and data packet loss—are considered.
- Loop detector is installed within communication range.

The control structure of the proposed system is presented in Fig. 1. The system is activated when a vehicle drives over the loop detector. At that moment, the system makes effort to establish communication with the vehicle. If the vehicle responds, then it is a CAV. Hence, the module 2 will be activated. Otherwise, it is a conventional uninformed vehicle. In that case, module 1 is activated. The details of the aforementioned modules are provided in the following:

- **Module 1: Conventional vehicle trajectory prediction.** This module is activated when a conventional vehicle drives pass the loop detector. The module first collects the vehicle’s (vehicle $n$) current state, signal timing, and the speed and trajectory information of the preceding vehicle (vehicle $n-1$). It then makes a prediction of the vehicle’s (vehicle $n$) future trajectory utilizing Intelligent Driver Model (IDM) Treiber et al., 2000 car-following model. The predicted speed profile and trajectory is stored in vehicle information for future use.

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Fig. 1. Control structure.
- **Module 2: Optimal controller for CAVs.** This module is activated when a CAV drives pass the loop detector. Similar to module 1, vehicle’s (vehicle $n$) initial state, trajectory information of the preceding vehicle (vehicle $n - 1$), and signal timing are collected. The module first calculates the expected terminal time at the stop line which can force the vehicle to keep the smallest headway to its preceding vehicle. Then the proposed optimal controller uses the expected terminal time and legal speed limit as a final condition to optimize the vehicle’s (vehicle $n$) future speed profile. The optimized speed profile is then transmitted to the CAV as speed advisory and is also stored in vehicle information for future use.

- **Module 3: Implementation.** The CAVs receive speed advisory from the proposed controller and adjust its speed accordingly.

### 3. Mathematical formulation

This section presents the formulations of module 1 and module 2 in detail. Module 1 is entailed in Section 3.1, and module 2 in Section 3.2. The acceleration profile of a conventional vehicle is calculated using Intelligent Driver Model (IDM) [Treiber et al., 2000]. The optimal speed profiles of CAV are solved using the Pontryagin Minimum Principle (PMP) [Pontryagin, 1962]. The proposed modeling and optimal controller have the following assumptions:

- The desired speed of conventional vehicles equals to the speed limit of the road segment. These vehicles travel at speed limit unless impeded by its preceding vehicle.
- The CAVs adopt the optimized speed advisory as desired speed. Car-following model no longer applies to CAVs.
- No turning and weaving maneuver.

Detailed formulations of prediction of conventional vehicles and optimal controller of CAVs are presented in the following two sections. Table 1 lists the indices and parameters utilized hereafter.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Indices and parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{1,n}(t)$</td>
<td>Location of vehicle $n$ (m), the coordinate is the distance from the loop detector</td>
</tr>
<tr>
<td>$x_{2,n}(t)$</td>
<td>Speed of vehicle $n$ (m/s)</td>
</tr>
<tr>
<td>$u_{n}(t)$</td>
<td>Acceleration of vehicle $n$ (m/s$^2$)</td>
</tr>
<tr>
<td>$u_{d}(t)$</td>
<td>Predicted acceleration of vehicle $n$ using IDM model in module 1 (m/s$^2$)</td>
</tr>
<tr>
<td>$k_{0}(t)$</td>
<td>Jerk of vehicle $n$ (m/s$^3$)</td>
</tr>
<tr>
<td>$\Delta x_{1,n}(t)$</td>
<td>Distance gap between vehicle $n$ and vehicle $n - 1$ (m)</td>
</tr>
<tr>
<td>$\Delta x_{2,n}(t)$</td>
<td>Desired minimum gap between vehicle $n$ and vehicle $n - 1$ (m)</td>
</tr>
<tr>
<td>$\Delta x_{2,u}(t)$</td>
<td>Speed difference between vehicle $n$ and vehicle $n - 1$ (m/s)</td>
</tr>
<tr>
<td>$s_0$</td>
<td>Jam distance (m)</td>
</tr>
<tr>
<td>$v_{lim}$</td>
<td>Legal speed limit of the road segment (m/s)</td>
</tr>
<tr>
<td>$v_{min}$</td>
<td>Minimum speed limit of vehicle $n$ (m/s)</td>
</tr>
<tr>
<td>$v_{d,n}$</td>
<td>Desired speed of vehicle $n$ (m/s)</td>
</tr>
<tr>
<td>$v_{i,n}$</td>
<td>Initial speed of vehicle $n$ at the loop detector in module 2 (m/s)</td>
</tr>
<tr>
<td>$v_{f,n}$</td>
<td>Terminal speed of vehicle $n$ at the stop line in module 2 (m/s)</td>
</tr>
<tr>
<td>$u_{a}$</td>
<td>Maximum acceleration of vehicle $n$ (m/s$^2$)</td>
</tr>
<tr>
<td>$u_{d}$</td>
<td>Minimum acceleration of vehicle $n$ (m/s$^2$)</td>
</tr>
<tr>
<td>$u_{d,n}$</td>
<td>Desired deceleration of vehicle $n$ (m/s$^2$)</td>
</tr>
<tr>
<td>$k_{d}$</td>
<td>Maximum jerk of vehicle $n$ (m/s$^3$)</td>
</tr>
<tr>
<td>$k_{d,n}$</td>
<td>Minimum jerk of vehicle $n$ (m/s$^3$)</td>
</tr>
<tr>
<td>$T$</td>
<td>Safe time headway (s)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Acceleration exponent</td>
</tr>
<tr>
<td>$L$</td>
<td>Distance from loop detector to stop line (m)</td>
</tr>
<tr>
<td>$t_{e,n}$</td>
<td>Initial time when vehicle $n$ reaches the loop detector (s)</td>
</tr>
<tr>
<td>$t_{e,n}^{-1}$</td>
<td>Terminal time when vehicle $n$ reaches the stop line (s)</td>
</tr>
<tr>
<td>$t_{e,n}$</td>
<td>Terminal time of vehicle $n - 1$ (s)</td>
</tr>
<tr>
<td>$t_{e,n}$</td>
<td>Candidate terminal time of vehicle $n$ (s)</td>
</tr>
<tr>
<td>$t_{e}^{-1}$</td>
<td>Earliest time of vehicle $n$ drives pass the stop line without considering its preceding vehicle and signal control (s)</td>
</tr>
<tr>
<td>$t_{s}$</td>
<td>Pre-set headway of two consecutive vehicles at the stop line (s)</td>
</tr>
<tr>
<td>$t_{r}$</td>
<td>The start time of red phase (s)</td>
</tr>
<tr>
<td>$e_{s,n}$</td>
<td>Spatial fuel consumption and emissions rate of vehicle $n$ (ml/m)</td>
</tr>
<tr>
<td>$e_{s,u}(t)$</td>
<td>Instantaneous fuel consumption and emissions rate of vehicle $n$ (ml/t)</td>
</tr>
<tr>
<td>$\beta_{1}, \beta_{2}$</td>
<td>Parameters of fuel consumption and emissions model</td>
</tr>
<tr>
<td>$R$</td>
<td>Duration of red phase (s)</td>
</tr>
<tr>
<td>$G$</td>
<td>Duration of green phase (s)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Collection of green phases</td>
</tr>
<tr>
<td>$t_{n}$</td>
<td>Time displacement of vehicle $n$ (s)</td>
</tr>
<tr>
<td>$\psi_{n}(x_{n}(t_{n}))$</td>
<td>Minimum safety distance of vehicle $n$ (m)</td>
</tr>
<tr>
<td>$L(x_{n}(t_{n}), u_{n}(t))$</td>
<td>Terminal cost of the optimal problem in module 2</td>
</tr>
<tr>
<td></td>
<td>Running cost of the optimal problem in module 2</td>
</tr>
</tbody>
</table>
3.1. Conventional vehicle trajectory prediction

As shown in Fig. 1, the module 1 is triggered when the system detects a conventional vehicle arriving at the loop detector. The inputs include instantaneous speed, entering time and signal phase and timing (SPaT) information from the traffic signal, together with the speed and trajectory information of its preceding vehicle. Microscopic car-following model called Intelligent Driver Model (IDM) is adopted to predict acceleration profile of conventional vehicles. The IDM is a well-accepted model for one-lane traffic. The predicted acceleration $u_n^p(t)$ can be formulated as follows:

$$u_n^p(t) = u_n \left[ 1 - \left( x_{2,n}(t) / p_n^m \right)^\delta - \left( \frac{\Delta x_{1,n}(x_{2,n}(t), \Delta x_{2,n}(t))}{\Delta x_{1,n}(t)} \right)^2 \right], \forall t \in [t_n^0, \infty)$$

(1)

According to the first assumption:

$$v_n^p = v_{\text{lim}}$$

(2)

The minimum desired gap is calculated as follows:

$$\Delta x_{1,n}(x_{2,n}(t), \Delta x_{2,n}(t)) = s_0 + \max \left( x_{2,n}(t) \cdot T + \frac{x_{2,n}(t) \cdot \Delta x_{2,n}(t)}{2 \sqrt{u_n \cdot u_n^p}}, 0 \right)$$

(3)

Signal timing could over-rule the car-following model when vehicles are near the intersection. As conventional vehicles approach the signalized intersection, there are two different situations:

- If the conventional vehicle and its preceding vehicle can enter the intersection in the same green phase, then use the calculated result as the acceleration of the conventional vehicle:

  $$u_n(t) = u_n^p(t), \forall t \in [t_n^0, \infty), \ t_n^p \in \xi$$

(4)

- If the conventional vehicle and its preceding vehicle cannot enter the intersection in the same green phase, then the conventional vehicle stops at the stop bar (VISSIM, 2007),

  $$t_s = \left| \frac{t_n^p}{R + G} \right| \cdot (R + G), \ \forall t_n^p \notin \xi$$

(5)

$$u_n(t) = \begin{cases} u_n^p, & \forall t \notin [t_n^0, \infty) \cap [t_R, t_R + R] \\ u_n \left[ 1 - \left( x_{2,n}(t) / p_n^m \right)^\delta - \left( \frac{\Delta x_{1,n}(x_{2,n}(t), \Delta x_{2,n}(t))}{\Delta x_{1,n}(t)} \right)^2 \right], & \forall t \in [t_n^0, \infty) \cap [t_R, t_R + R] \end{cases}$$

(6)

$$\Delta x_{1,n}(x_{2,n}(t), \Delta x_{2,n}(t)) = s_0 + \max \left( x_{2,n}(t) \cdot T + \frac{x_{2,n}(t)^2}{2 \sqrt{u_n \cdot u_n^p}}, 0 \right)$$

(7)

3.2. Optimal controller for CAVs

As shown in Fig. 1, the optimal controller (module 2) is triggered when there is an CAV detected by loop detector. The inputs of the optimal controller are instantaneous speed, entering time and signal phase and timing (SPaT) information from the traffic signal, together with the trajectory information of the preceding vehicle. The objective is to minimize total fuel consumption and emissions and maximize comfort while maintaining the throughput at its optimum level. To reduce the computational burden, the problem is constructed in PMP structure and solved using a numerical PMP approach (Pontryagin, 1962). The output generated by optimal controller is an optimal acceleration profile, it is the most fuel efficient among all the acceleration profiles satisfy the optimal mobility constraint.

3.2.1. State explanation

For an individual CAV indexed by $n$, the system state vector $x_n(t)$ can be defined as follows:

$$x_n(t) = [x_{1,n}(t), x_{2,n}(t)]^T$$

(8)

The state dynamics is shown as follows:

$$\dot{x}_n(t) = f(x_n(t), u_n(t)) = [x_{2,n}(t), u_n(t)]^T$$

(9)

with $u_n(t) = u_n(t)$ denotes the control input, which in this case is the acceleration of the vehicle $n$.

3.2.2. Cost function

The cost function is defined as follows:
\[ J = \varphi(x_n(t_n^0)) + \int_{t_n^0}^{t_n^1} L(x_n(t), u_n(t)) dt \]  

where

\[ t_n^{L_C} = \max(t_{n-1}^{L_C} + \delta_n, t_n^0) \]

\[ t_n^{L_C} = t_n^0 + \frac{L - (\varphi_{\text{lim}}^2 - x_{2,n}(t_n^0)^2)/2\bar{u}_n}{\varphi_{\text{lim}}} + \frac{\varphi_{\text{lim}} - x_{2,n}(t_n^0)}{\ddot{u}_n} \]

\[ t_n^L \begin{cases} 
  t_n^{L_C} & \forall t_n^{L_C} \in \mathbb{Z} \\
  \left\lfloor t_n^{L_C} \right\rfloor \cdot (R + G) + R & \forall t_n^{L_C} \notin \mathbb{Z}
\end{cases} \]

where \( \varphi(x_n(t_n^0)) \) is the terminal cost and \( L(x_n(t), u_n(t)) \) is the running cost. The terminal cost is formulated as:

\[ \varphi(x_n(t_n^0)) = w_1(x_{1,n}(t_n^0) - L)^2 + w_2(x_{2,n}(t_n^0) - v_n^0)^2, w_1 \in \mathbb{R}^+, w_2 \in \mathbb{R}^+ \]

where \( w_1 \) and \( w_2 \) are large positive numbers which ensure a constraint on vehicle’s final state. The terminal cost \( \varphi(x_n(t_n^0)) \) ensures the optimized CAV can enter the intersection on time at a preferred speed. In order to maximize throughput, the terminal speed of all CAVs is set equal to the legal speed limit of the road segment (Zhou et al., 2017). This is the first measure to prioritize mobility.

\[ v_n^0 = \varphi_{\text{lim}} \]

The running cost is formulated as:

\[ L(x_n(t), u_n(t)) = w_3 \cdot g_{s,n} + \frac{1}{2} u_n(t)^2, w_3 \in \mathbb{R}^+ \]

\[ g_{s,n} = g_{t,n}(t)/x_{2,n}(t) \]

where \( w_3 \) is a weighting factor for fuel efficiency, and the unit is m^3/(ml s^4) (Wang et al., 2014). The quadratic term of acceleration is the comfort consideration.

Fuel consumption and emissions model developed by Akcelik is adopted (Akcelik, 1989).

\[ g_{t,n}(t) = \sum_{j=0}^{3} \gamma_j \cdot x_{2,n}(t)^j + \beta_1 \cdot x_{2,n}(t) \cdot u_n(t) + \beta_2 \cdot x_{2,n}(t) \cdot u_n(t)^2 \cdot \chi(u_n(t)) \]

where \( \chi(u_n(t)) \) is a Heaviside function of acceleration:

\[ \chi(u_n(t)) = \begin{cases} 
  1 & u_n(t) \geq 0 \\
  0 & u_n(t) < 0
\end{cases} \]

### 3.2.3. Conditions and constraints

The initial conditions are:

\[ x_{1,n}(t_n^0) = 0, x_{2,n}(t_n^0) = v_n^0 \]

When solving for the optimal vehicle speed profile, the aforementioned cost function has the following constraints.

**Speed constraint:** For the consideration of mobility and the feasibility of adjusting speed in a mixed traffic scenario, speed is designed to change in a designated range. In this study, maximum speed is the speed limit and minimum speed is a user predetermined value, according to the previous research, the vehicle fuel efficiency decrease greatly below 4.47 m/s (10 mile/h) Barth et al., 2011, so the nearest integral value 4 m/s (14.4 km/h) is adopted as the minimum speed. This constraint can be specified as:

\[ v_n = \{ x_{2,n} | v_n^0 \leq x_{2,n}(t) \leq \varphi_{\text{lim}}, \forall t \in [t_n^0, t_n^1] \} \]

**Acceleration constraint:** To ensure that all acceleration solutions are feasible provided the engine maximum power and brake condition. This acceleration range is solely for the eco-driving optimal controller which is one of the many applications that are installed on connected and automated vehicles and could be over-ruled by collision prevention applications. In other words, vehicles are able to brake much faster than the maximum deceleration when safety hazard arises. This constraint can be expressed as a permissible set of acceleration:

\[ u_n = \{ u_n^0 | u_n^0 \leq u_n(t) \leq \ddot{u}_n, \forall t \in [t_n^0, t_n^1] \} \]
Jerk is bounded for better comfort and can be calculated as follows.

\[
k_n(t) = \frac{\partial u_n(t)}{\partial t}
\]

(23)

\[
\kappa_n = \{k_n | k_n^0 \leq k_n(t) \leq k_n, \forall t \in [t_n^0, t_n^n]\}
\]

(24)

Safety constraint: As described in Newell car-following model, a distance displacement and time displacement should be guaranteed to maintain safety distance between consecutive vehicles (Newell, 2002). The vehicles with unsafe trajectory would disable their eco-drive function and drive following a regular car following model to maximize passengers’ comfort.

\[
x_{1,n}(t + \tau_n) \leq x_{1,n-1}(t) - d_n
\]

(25)

3.2.4. Solution based on Pontryagin’s Minimum Principle

The aforementioned optimal control problem is solved using PMP approach. The key is to find Hamiltonian function \( \mathcal{H} \):

\[
\mathcal{H}(x, u, \lambda, t) = \lambda^T \cdot f(x, u, t) + L(x, u, t)
\]

(26)

where \( \lambda \) denotes the co-state of the state \( x \), which is the extra cost of \( f \) as a result of a small change \( \partial x \) in the state \( x \). According to the PMP, all optimal control should fall into the admissible set \( \mathcal{U} \), the necessary condition for optimal control \( u^* \) can be derived as follows:

\[
\mathcal{H}(x^*, u^*, \lambda^*, t) = \mathcal{H}(x^*, u^*, \lambda^*, t), \forall u \in \mathcal{U}, t \in [t_n^0, t_n^n]
\]

(27)

This necessary condition can be expressed alternatively as follows:

\[
0 = \frac{\partial \mathcal{H}}{\partial u}
\]

(28)

\[
\dot{\lambda} = \frac{\partial \mathcal{H}}{\partial x}
\]

(29)

\[
\ddot{x} = \frac{\partial \mathcal{H}}{\partial \lambda}
\]

(30)

The Eqs. (28) and (29) are used to solve the optimal control \( u^* \) and Eq. (30) is same as state dynamics. Substituting the Hamiltonian function with the cost function \( L(x_n(t), u_n(t)) \):

\[
\mathcal{H}_n = \lambda_1 \cdot x_{2,n}(t) + \lambda_2 \cdot u_n(t) + w_3 \cdot x_0 \cdot x_{2,n}(t)^{-1} + \alpha_1 + \alpha_2 \cdot x_{2,n}(t) + \alpha_3 \cdot x_{2,n}(t)^2 + \ldots + \beta_1 \cdot u_n(t) + \beta_2 \cdot u_n(t)^2 - \gamma(u_n(t)) + \frac{1}{2} u_n(t)^2
\]

(31)

Apply Eq. (31) into Eq. (28), the control law of the proposed optimal control can be found:

\[
\begin{cases}
    u_n(t) = -\frac{\lambda_2 + w_3\beta_1}{\lambda_1 + w_2\beta_2}, & \forall \lambda_2 \leq -w_3\beta_1 \\
    u_n(t) = -\frac{\lambda_2 + w_3\beta_1}{\lambda_1 + w_2\beta_2}, & \forall \lambda_2 > -w_3\beta_1
\end{cases}
\]

(32)

Similarly, the Eq. (29) provides:

\[
\dot{\lambda}_1 = -\frac{\partial \mathcal{H}_n}{\partial x_{1,n}} = 0 \Rightarrow \lambda_1 = C
\]

(33)

\[
\dot{\lambda}_2 = -\frac{\partial \mathcal{H}_n}{\partial x_{2,n}} = -\lambda_1 + w_3(x_0 \cdot x_{2,n}(t)^{-2} - \alpha_2 - 2\alpha_3 \cdot x_{2,n}(t))
\]

(34)

where \( C \) is a constant to be determined.

In order to enforce the desired terminal state \( \phi(x_n(t_n^n)) \) to ensure optimal throughput at the intersection, the co-state \( \lambda \) should satisfy the following condition:

\[
\lambda(t_n^n) = \frac{\partial \phi(x_n(t_n^n))}{\partial x_n}
\]

(35)

which gives:

\[
\dot{\lambda}_1(t_n^n) = 2w_1(x_{1,n}(t_n^n) - L)
\]

(36)

\[
\dot{\lambda}_2(t_n^n) = 2w_2(x_{2,n}(t_n^n) - v_{lim})
\]

(37)
3.2.5. Iterative PMP solving process

To solve the aforementioned problem, a numerical solution is adopted (Hoogendoorn et al., 2016). The main idea is to iteratively find state $x$ forward in time and then co-state $\lambda$ backward in time. The algorithm is summarized as follows:

1. Choose a weighting factor $0 < \gamma < 1$ for smoothly updating the co-state, and pre-set the error tolerance $\varepsilon_{\text{max}}$ of iteration;
2. Initialize the co-state $\lambda^{(0)}(t) = 0$ for $t \in [t^n_0, t^n_L]$;
3. Solve the state dynamic equations subject to first three constraints (including jerk, acceleration and speed) forward in time for $x^{(m)}(t)$ with co-state $\lambda^{(m-1)}(t)$ computed from the previous iteration;
4. Solve the co-state dynamic equations backward in time for $\lambda^{(m)}(t)$ using $x^{(m)}(t)$ from the previous step;

Fig. 2. Illustration of the eco-driving system test network.

Table 2
Attributes in the simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>VISSIM</strong></td>
<td></td>
</tr>
<tr>
<td>Distance from loop detector to intersection (m)</td>
<td>300</td>
</tr>
<tr>
<td>Simulation time horizon (s)</td>
<td>1200</td>
</tr>
<tr>
<td>Warming up time (s)</td>
<td>180</td>
</tr>
<tr>
<td>Cycle of signal timing (s)</td>
<td>60</td>
</tr>
<tr>
<td>Duration of red light (s)</td>
<td>25</td>
</tr>
<tr>
<td>Duration of green light (s)</td>
<td>35</td>
</tr>
<tr>
<td>Saturation flow rate (veh/h)</td>
<td>1830</td>
</tr>
<tr>
<td><strong>Conventional vehicle trajectory prediction</strong></td>
<td></td>
</tr>
<tr>
<td>Desired speed (km/h)</td>
<td>72.5</td>
</tr>
<tr>
<td>Minimum speed (km/h)</td>
<td>0</td>
</tr>
<tr>
<td>Safe time headway (s)</td>
<td>1.6</td>
</tr>
<tr>
<td>Maximum acceleration (m/s²)</td>
<td>3.5</td>
</tr>
<tr>
<td>Desired deceleration (m/s²)</td>
<td>-2.8</td>
</tr>
<tr>
<td>Acceleration exponent</td>
<td>4</td>
</tr>
<tr>
<td>Jam distance (m)</td>
<td>5</td>
</tr>
<tr>
<td><strong>Optimal controller for CAVs</strong></td>
<td></td>
</tr>
<tr>
<td>Maximum acceleration (m/s²)</td>
<td>3.5</td>
</tr>
<tr>
<td>Minimum acceleration (m/s²)</td>
<td>-4</td>
</tr>
<tr>
<td>Speed limit (km/h)</td>
<td>72.5</td>
</tr>
<tr>
<td>Minimum speed (km/h)</td>
<td>14.4</td>
</tr>
<tr>
<td>Jerk (m/s³)</td>
<td>10</td>
</tr>
<tr>
<td>Time step (s)</td>
<td>0.1</td>
</tr>
<tr>
<td>Headway of two consecutive vehicles at the stop line (s)</td>
<td>2.0</td>
</tr>
<tr>
<td>Factor $w_1$</td>
<td>10</td>
</tr>
<tr>
<td>Factor $w_2$</td>
<td>100</td>
</tr>
<tr>
<td>Factor $w_3$</td>
<td>1</td>
</tr>
<tr>
<td>$\varepsilon_{\text{max}}$</td>
<td>50</td>
</tr>
</tbody>
</table>
(5) Update the co-state $\Lambda^{(m)}$ based on $\lambda^{(m)}$ and $\Lambda^{(m-1)}$ from the previous iteration,

$$
\Lambda^{(m)} = (1 - \gamma)\Lambda^{(m-1)} + \gamma \cdot \lambda^{(m)}
$$

(6) Stop the iteration if $\|\Lambda^{(m)} - \lambda^{(m)}\| < \epsilon_{\text{max}}$, otherwise set $m = m + 1$ and go back to step 3.

4. Simulation evaluation

The proposed eco-driving system is evaluated through a microscopic simulation. The test network is a hypothetical intersection, as shown in Fig. 2. Each travel direction is with single travel lane. Therefore, there is no weaving and turning maneuver allowed in the simulation. Traffic is generated on EB only. One loop detector is installed on the EB approach at 300 m upstream of the intersection. The network has been calibrated according to Highway Capacity Manual 2010 (Manual, 2010). Saturation flow rate has been checked, as shown in Table 2.

An integrated simulation platform is constructed using multiple software tools, including VISSIM, Matlab and Excel VBA. The Excel VBA is the master control program and responsible for the communication between VISSIM and Matlab. VISSIM simulates traffic and generates input for Matlab control/prediction algorithm (as the central controller in Fig. 2). In order to truly represent the real world where connected and automated vehicle could be impeded by its preceding vehicle and cannot drive at its desired speed, the optimized speed trajectory is written back to the corresponding CAV in VISSIM as “Desired Speed”, so there may exist an inconsistency between actual speed and optimized speed.

The proposed optimal controller was evaluated with a sensitivity analysis on two factors, including congestion level and CAV market penetration rate (MPR). At least ten different random seeds have been simulated for each scenario. Sample size has been checked. More random seeds were tested if the sample size was not sufficient. Measurements of effectiveness adopted are throughput, fuel consumption and CO2 emissions. To fairly confirm the benefits on fuel consumption and emissions, the fuel consumption and emissions computation model used in the cost function is not applied in the evaluation. Instead, another well-accepted model called VT-Micro (Rakha et al., 2004) is adopted for the evaluation purpose.

(a) Optimal controller computation time of various prediction horizons and time step sizes

(b) Convergence curve of iterative PMP

Fig. 3. Optimal controller computation time and convergence curve of iterative PMP.
The following scenarios are tested in this research:

- **Non-automation baseline [Base]**: The baseline scenario of this research is defined as when CAV market penetration rate is zero. This is when proposed algorithm has no effect on traffic. Because various congestion levels are tested in the sensitivity study, there is a baseline scenario for each congestion level.

- **Proposed controller**: In this scenario, part/all of the simulated vehicles are CAV. All CAVs are controlled by the proposed eco-drive controller.

- **State-of-the-art Eco-drive [GlidePath]**: In this scenario, part/all of the simulated vehicles are CAV. All CAVs are controlled by the state-of-the-art eco-drive controller. In this research, the state-of-the-art eco-drive controller refers to the GlidePath eco-drive system which is recently developed by the Federal Highway Administration (FHWA) \cite{FHWA,2016}.

![Image of graphs showing hourly traffic volumes, throughput benefits, and average fuel economy improvements.](image-url)

**Fig. 4.** Improvements of throughput, fuel efficiency and emission reduction.
The following assumptions were made for the simulation evaluation:

- Vehicles composition is 100% mid-sized vehicle with the same size and dynamic characteristics;
- The gradient of the road segment is zero;
- Poisson distribution is used to model the vehicle arrival pattern.

Table 2 shows all the attributes set in the simulation.

4.1 Simulation results

The simulation results are presented in this section. They confirm that the proposed eco-driving system can achieve the aforementioned objectives: reduces fuel consumption and emissions while maintains the throughput of an isolated signalized intersection at its optimal condition. As shown in the results, the proposed system could save fuel by up to 58%, reduce emissions by up to 33% and improve throughput by up to 11%.
The computational time for the optimization is 0.94 s, given a 50 s optimization time horizon and 0.2-s time step. Computation time can be further reduced with greater time step size and shorter optimization time horizon. A sensitivity analysis on optimization time horizon and time step size is presented in Fig. 3(a). The proposed system can potentially be used in real-time.

Fig. 3(b) shows a convergence curve of iterative PMP with 50 s optimization time horizon and 0.2-s time step, error tolerance $\epsilon_{\text{max}}$ is set to 50. The error quickly drops after less than 90 iterations. The total number of iterations is 918.

4.1.1. Comparison against Non-automation baseline

Fig. 4(a) and (b) shows the throughput benefits under various congestion levels and market penetration rates (MPR) of CAVs. The proposed system can effectively improve throughput when the intersection is saturated (V/C = 1.0) or oversaturated (V/C = 1.2). Under these two levels of saturation, more throughput benefits were achieved with higher MPR of CAV. The benefit under oversaturated condition is higher than under saturated condition at the same MPR level. Significant benefits are observed for all levels of MPR. This phenomenon indicates the proposed system is able to make a difference with low MPR of CAV and can potentially be applied in the real world traffic as long as CAV is introduced. It also confirms that the proposed eco-driving system overcomes the problem of existing systems which have an adverse effect on throughput. To be noted, the proposed system generates no benefit on throughput under non-saturated condition (V/C = 0.5). This makes sense because non-congested roads can easily handle all the vehicles and thus is with no room for improvement.

Fig. 4(c) and (d) shows fuel consumption benefits under various congestion and MPR levels. Significant benefits are observed as long as there is CAV. The benefits grow with the MPR of CAV. The growing trend levels off as MPR reaches

![figures](image-url)
40%. In addition, fuel consumption benefits grow with congestion level. The reason is that CAVs can have greater impacts on the following conventional vehicles under higher saturation condition. As shown in Fig. 5(a), compared with the simulation result under saturated condition (Fig. 5(a)-iv), there exists a heavier traffic on the road segment under oversaturated condition (Fig. 5(a)-vi), the following conventional vehicles have more similar spatial-temporal trajectories to the preceding CAV. In other words, the conventional vehicles can be better “controlled” with heavier traffic.

Fig. 4(e) and (f) shows CO₂ emissions benefits. Overall, emissions reduction rises with MPR and congestion level. Fig. 5(a) shows the spatial-temporal trajectories with MPR = 30% under different saturation levels. On the left, the trajectory, named as “predicted trajectory”, is from the computation of conventional vehicle trajectory prediction and optimized CAV trajectory. On the right, the trajectory, named as “simulation trajectory”, is the “real” trajectory collected from simulation with CAVs trying to drive at its instructed speed. The comparison between predicted trajectories and simulation
trajectories confirms the accuracy of the proposed module 1. This is very important because the optimal controller for CAVs need the trajectory of its preceding vehicle as input. In addition, the simulation results show that the actual spatial-temporal trajectories of CAVs exist deviations from their advisory trajectories. The aforementioned significant benefits are achieved despite such deviation. It proves the robustness of the proposed eco-driving system.

Fig. 5(b) shows the spatial-temporal trajectories of simulation results under saturated condition. As the MPR of CAVs increases, fewer conventional vehicles need to queue and waiting for green phase. This growing trend is very obvious and significant when MPR is less than or equal to 30%. When the MPR is more than 30%, this trend becomes less obvious gradually. This is consistent with the findings from fuel consumption and emissions benefits.

Fig. 5(c) shows the spatial-temporal trajectories of simulation results under oversaturated condition. Compared with the saturated scenarios, a smaller headway is observed with the same MPR and leads to a better control effect on conventional

(c) Spatial-temporal trajectories of simulation results (V/C=1.2)

Fig. 5 (continued)
vehicles. It is also observed that the shock wave caused by signal control can be smoothed out by the proposed eco-driving system. It explains the throughput benefit found previously.

4.1.2. Comparison against the State-of-the-art Eco-drive

The proposed eco-drive system is compared against the state-of-the-art eco-drive called “GlidePath” which is recently developed by the Federal Highway Administration (FHWA). The simulation results are presented in this section. They confirm that the proposed eco-driving system outperforms the state-of-the-art eco-drive system. As shown in Fig. 6, the proposed system could save fuel by up to 38%, reduce emissions by up to 25% and improve throughput by up to 9%.

Fig. 7 shows the spatial-temporal trajectories of “GlidePath” under different MPR of CAVs when v/c = 1.0. On the left, the trajectory, named as “predicted trajectory”, is from the computation of conventional vehicle trajectory prediction and optimized CAV trajectory. On the right, the trajectory, named as “simulation trajectory”, is the “real” trajectory collected from
simulation with CAVs trying to drive at its instructed speed. From the figures, there exist big differences between planned trajectories and real trajectories. As a result, larger headway and greater shockwaves are observed. Those leads directly to a loss in throughput, fuel efficiency and increase in emission.

The main difference between the proposed system and the GlidePath is that the proposed system actively step into “control” the entire traffic. The leading CAVs’ deceleration and following CAVs’ acceleration not only tighten platoon but also leave human driven vehicles with less room to be stochastic. This design keeps the traffic calm and ensures benefit in throughput, fuel efficiency and emission.

5. Conclusions and future research

This research proposed an eco-driving system for an isolated signalized intersection under partially Connected and Automated Vehicles (CAV) environment. The proposed system improves fuel efficiency and reduces emissions while maintaining mobility at its optimal level. It overcomes the shortcomings of the existing eco-driving systems and is able to: (i) function in partially connected and automated vehicles environment which makes the proposed system ready for real-world implementation; (ii) improve fuel efficiency and reduce emissions while does no harm to throughput if not benefit it. The proposed eco-driving system was formulated as an optimal control problem which was solved using Pontryagin’s Minimum Principle. The evaluation of the control system showed:

- Throughput benefits range from 0.88% to 7.06% under saturated condition and from 1.74% to 10.80% under oversaturated condition. The variations are caused by the market penetration rate of CAVs. No adverse effect is observed when traffic is under-saturated.
- Fuel consumption benefits range from 2.02% to 8.65% under non-saturated condition, from 9.95% to 44.02% under saturated condition and from 13.13% to 58.01% under oversaturated condition. The variations are also caused by the market penetration rate of CAVs.
- The emissions benefits range from 1.97% to 7.29% under non-saturated condition, from 8.22% to 27.90% under saturated condition and from 10.56% to 33.26% under oversaturated condition. The variations are also caused by the market penetration rate of CAVs.
- The proposed system outperforms the state-of-the-art eco-drive system, as it is less affected by stochastic human-driven vehicles.
- The proposed system is beneficial as long as there is a connected and automated vehicle in the traffic system. The benefits grow with the market penetration rate of connected and automated vehicles until they level off at about 40% MPR. This indicates that the proposed system can be implemented even with a low market penetration rate of connected and automated vehicles and can be applied in real-world traffic faster than most of the other connected and automated vehicles applications.

Fig. 7. Spatial-temporal trajectories of GlidePath.
• The proposed eco-driving system is able to smooth out the shock wave caused by signal controls.
• The proposed system is robust over the impedance from conventional vehicles and randomness of traffic.
• The computational time for optimization is up to 0.94 s, given a 50 s prediction time horizon and a 0.2 s update horizon. The proposed system can potentially be used in real-time.

Future research should also consider more complex traffic scene, for instance, multiple lanes with lane-change behavior. In addition, future study should also upgrade the eco-driving system to consider the progression along the interested corridor. Furthermore, a cooperative system of adaptive signal control and the eco-driving system could be developed to further advance the proposed system. At the same time of upgrading the system, efforts should be made to reduce computational complexity for potential real-time applications. Making simplifications on the objective function in order to get analytical solutions is one important option.

Acknowledgements

This research was in part supported by the Global Research Laboratory Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning (2013K1A1A2A02078326), and by the Korea Agency for Infrastructure Technology Advancement (KAIA) grant funded by the Korea Government (MOLIT) on the Development and Verification of Signal Operation Algorithms in Local Intersection Network utilizing V2X Communication Infrastructure (No. 15T1RP-C105654-01).

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